JOURNAL OF FORMALIZED MATHEMATICS
Volume 4, Released 1992, Published 2003
Inst. of Computer Science, Univ. of Białystok

Definitions of Petri Net. Part II

Waldemar Korczyński
Pedagogical University
Kielce

Summary. In the paper an equational definition of Petri net is given.

MML Identifier: E_SIEC.

WWW: http://mizar.org/JFM/Vol4/e_siec.html

The articles [3], [1], [4], and [2] provide the notation and terminology for this paper.

In this paper x, y, X, Y are sets.

We introduce G-net structures which are extensions of 1-sorted structure and are systems
⟨ a carrier, an entrance, an escape ⟩,
where the carrier is a set and the entrance and the escape are binary relations.

Let N be a 1-sorted structure. The functor echaos(N) yields a set and is defined by:

(Def. 1) $\mathrm{echaos}(N) = (\text{the carrier of } N) \cup \{\text{the carrier of } N\}$.

Let I_1 be a G-net structure. We say that I_1 is GG if and only if the conditions (Def. 2) are satisfied.

(Def. 2)(i) The entrance of $I_1 \subseteq [:\text{the carrier of } I_1, \text{ the carrier of } I_1:]$,

(ii) the escape of $I_1 \subseteq [:\text{the carrier of } I_1, \text{ the carrier of } I_1:]$,

(iii) $(\text{the entrance of } I_1) \cdot (\text{the entrance of } I_1) = \text{the entrance of } I_1$,

(iv) $(\text{the entrance of } I_1) \cdot (\text{the escape of } I_1) = \text{the entrance of } I_1$,

(v) $(\text{the escape of } I_1) \cdot (\text{the escape of } I_1) = \text{the escape of } I_1$, and

(vi) $(\text{the escape of } I_1) \cdot (\text{the entrance of } I_1) = \text{the escape of } I_1$.

Let us observe that there exists a G-net structure which is GG.

A G-net is a GG G-net structure.

Let I_1 be a G-net structure. We say that I_1 is EE if and only if the conditions (Def. 3) are satisfied.

(Def. 3)(i) $(\text{The entrance of } I_1) \cdot ((\text{the entrance of } I_1) \setminus \mathrm{id}_{\text{the carrier of } I_1}) = \emptyset$, and

(ii) $(\text{the escape of } I_1) \cdot ((\text{the escape of } I_1) \setminus \mathrm{id}_{\text{the carrier of } I_1}) = \emptyset$.

Let us mention that there exists a G-net structure which is EE.

Let us note that there exists a G-net structure which is strict, GG, and EE.

An E-net is an EE GG G-net structure.

In the sequel N denotes an E-net.

Next we state several propositions:

(1) Let R, S be binary relations. Then $\langle X, R, S\rangle$ is an E-net if and only if the following conditions are satisfied:
$R \subseteq [:X, X:]$ and $S \subseteq [:X, X:]$ and $R \cdot R = R$ and $R \cdot S = R$ and $S \cdot S = S$ and $S \cdot R = S$ and $R \cdot (R \setminus \mathrm{id}_X) = \emptyset$ and $S \cdot (S \setminus \mathrm{id}_X) = \emptyset$.

(2) $\langle X, \emptyset, \emptyset \rangle$ is an E-net.

(3) $\langle X, \mathrm{id}_X, \mathrm{id}_X \rangle$ is an E-net.

(4) $\langle \emptyset, \emptyset, \emptyset \rangle$ is an E-net.

(8)[1] $\langle X, \mathrm{id}_{X \setminus Y}, \mathrm{id}_{X \setminus Y} \rangle$ is an E-net.

(9) $\mathrm{echaos}(N) \neq \emptyset$.

The strict E-net empty_e is defined as follows:

(Def. 4) $\mathrm{empty}_e = \langle \emptyset, \emptyset, \emptyset \rangle$.

Let us consider X. The functor $\mathrm{Tempty}_e(X)$ yielding a strict E-net is defined as follows:

(Def. 5) $\mathrm{Tempty}_e(X) = \langle X, \mathrm{id}_X, \mathrm{id}_X \rangle$.

The functor $\mathrm{Pempty}_e(X)$ yields a strict E-net and is defined as follows:

(Def. 6) $\mathrm{Pempty}_e(X) = \langle X, \emptyset, \emptyset \rangle$.

Next we state two propositions:

(11)[2] The carrier of $\mathrm{Tempty}_e(X) = X$ and the entrance of $\mathrm{Tempty}_e(X) = \mathrm{id}_X$ and the escape of $\mathrm{Tempty}_e(X) = \mathrm{id}_X$.

(12) The carrier of $\mathrm{Pempty}_e(X) = X$ and the entrance of $\mathrm{Pempty}_e(X) = \emptyset$ and the escape of $\mathrm{Pempty}_e(X) = \emptyset$.

Let us consider x. The functor $\mathrm{Psingle}_e(x)$ yields a strict E-net and is defined by:

(Def. 7) $\mathrm{Psingle}_e(x) = \langle \{x\}, \mathrm{id}_{\{x\}}, \mathrm{id}_{\{x\}} \rangle$.

The functor $\mathrm{Tsingle}_e(x)$ yields a strict E-net and is defined as follows:

(Def. 8) $\mathrm{Tsingle}_e(x) = \langle \{x\}, \emptyset, \emptyset \rangle$.

Next we state three propositions:

(13) The carrier of $\mathrm{Psingle}_e(x) = \{x\}$ and the entrance of $\mathrm{Psingle}_e(x) = \mathrm{id}_{\{x\}}$ and the escape of $\mathrm{Psingle}_e(x) = \mathrm{id}_{\{x\}}$.

(14) The carrier of $\mathrm{Tsingle}_e(x) = \{x\}$ and the entrance of $\mathrm{Tsingle}_e(x) = \emptyset$ and the escape of $\mathrm{Tsingle}_e(x) = \emptyset$.

(15) $\langle X \cup Y, \mathrm{id}_X, \mathrm{id}_X \rangle$ is an E-net.

Let us consider X, Y. The functor $\mathrm{PTempty}_e(X, Y)$ yielding a strict E-net is defined by:

(Def. 9) $\mathrm{PTempty}_e(X, Y) = \langle X \cup Y, \mathrm{id}_X, \mathrm{id}_X \rangle$.

One can prove the following propositions:

(16)(i) (The entrance of $N) \setminus \mathrm{id}_{\mathrm{dom}(\text{the entrance of } N)} =$ (the entrance of $N) \setminus \mathrm{id}_{\text{the carrier of } N}$,

(ii) (the escape of $N) \setminus \mathrm{id}_{\mathrm{dom}(\text{the escape of } N)} =$ (the escape of $N) \setminus \mathrm{id}_{\text{the carrier of } N}$,

(iii) (the entrance of $N) \setminus \mathrm{id}_{\mathrm{rng}(\text{the entrance of } N)} =$ (the entrance of $N) \setminus \mathrm{id}_{\text{the carrier of } N}$, and

(iv) (the escape of $N) \setminus \mathrm{id}_{\mathrm{rng}(\text{the escape of } N)} =$ (the escape of $N) \setminus \mathrm{id}_{\text{the carrier of } N}$.

(17) $\mathrm{CL}(\text{the entrance of } N) = \mathrm{CL}(\text{the escape of } N)$.

[1] The propositions (5)–(7) have been removed.
[2] The proposition (10) has been removed.

(18)(i) rng(the entrance of N) $= \mathrm{rng}\,\mathrm{CL}$(the entrance of N),

(ii) rng(the entrance of N) $= \mathrm{dom}\,\mathrm{CL}$(the entrance of N),

(iii) rng(the escape of N) $= \mathrm{rng}\,\mathrm{CL}$(the escape of N),

(iv) rng(the escape of N) $= \mathrm{dom}\,\mathrm{CL}$(the escape of N), and

(v) rng(the entrance of N) $= \mathrm{rng}$(the escape of N).

(19)(i) dom(the entrance of N) $\subseteq$ the carrier of N,

(ii) rng(the entrance of N) $\subseteq$ the carrier of N,

(iii) dom(the escape of N) $\subseteq$ the carrier of N, and

(iv) rng(the escape of N) $\subseteq$ the carrier of N.

(20)(i) (The entrance of N) $\cdot$ ((the escape of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$, and

(ii) (the escape of N) $\cdot$ ((the entrance of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$.

(21)(i) ((The entrance of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) \cdot$ ((the entrance of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$,

(ii) ((the escape of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) \cdot$ ((the escape of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$,

(iii) ((the entrance of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) \cdot$ ((the escape of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$, and

(iv) ((the escape of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) \cdot$ ((the entrance of N) $\setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$.

Let us consider N. The functor $\mathrm{Places}_e(N)$ yielding a set is defined by:

(Def. 10) $\mathrm{Places}_e(N) = \mathrm{rng}$(the entrance of N).

Let us consider N. The functor $\mathrm{Transitions}_e(N)$ yields a set and is defined by:

(Def. 11) $\mathrm{Transitions}_e(N) =$ (the carrier of N) $\setminus \mathrm{Places}_e(N)$.

Next we state three propositions:

(22) $\mathrm{Places}_e(N)$ misses $\mathrm{Transitions}_e(N)$.

(23) If $\langle x, y\rangle \in$ the entrance of N or $\langle x, y\rangle \in$ the escape of N and if $x \neq y$, then $x \in \mathrm{Transitions}_e(N)$ and $y \in \mathrm{Places}_e(N)$.

(24) (The entrance of N) $\setminus \mathrm{id}_{\text{the carrier of } N} \subseteq [:\mathrm{Transitions}_e(N), \mathrm{Places}_e(N):]$ and (the escape of N) $\setminus \mathrm{id}_{\text{the carrier of } N} \subseteq [:\mathrm{Transitions}_e(N), \mathrm{Places}_e(N):]$.

Let us consider N. The functor $\mathrm{Flow}_e(N)$ yielding a binary relation is defined by:

(Def. 12) $\mathrm{Flow}_e(N) =$ ((the entrance of N)$^{\smile} \cup$ the escape of N) $\setminus \mathrm{id}_{\text{the carrier of } N}$.

Next we state the proposition

(25) $\mathrm{Flow}_e(N) \subseteq [:\mathrm{Places}_e(N), \mathrm{Transitions}_e(N):] \cup [:\mathrm{Transitions}_e(N), \mathrm{Places}_e(N):]$.

Let us consider N. We introduce $\mathrm{places}_e(N)$ as a synonym of $\mathrm{Places}_e(N)$. We introduce $\mathrm{transitions}_e(N)$ as a synonym of $\mathrm{Transitions}_e(N)$.

Let us consider N. The functor $\mathrm{pre}_e(N)$ yields a binary relation and is defined by:

(Def. 15)[3] $\mathrm{pre}_e(N) =$ (the entrance of N) $\setminus \mathrm{id}_{\text{the carrier of } N}$.

The functor $\mathrm{post}_e(N)$ yielding a binary relation is defined as follows:

(Def. 16) $\mathrm{post}_e(N) =$ (the escape of N) $\setminus \mathrm{id}_{\text{the carrier of } N}$.

We now state the proposition

[3] The definitions (Def. 13) and (Def. 14) have been removed.

(28)[4] $\mathrm{pre}_e(N) \subseteq [:\mathrm{transitions}_e(N), \mathrm{places}_e(N):]$ and $\mathrm{post}_e(N) \subseteq [:\mathrm{transitions}_e(N), \mathrm{places}_e(N):]$.

Let us consider N. The functor $\mathrm{shore}_e(N)$ yields a set and is defined by:

(Def. 17) $\mathrm{shore}_e(N) =$ the carrier of N.

The functor $\mathrm{prox}_e(N)$ yields a binary relation and is defined by:

(Def. 18) $\mathrm{prox}_e(N) = ((\text{the entrance of } N) \cup (\text{the escape of } N))^\smile$.

The functor $\mathrm{flow}_e(N)$ yields a binary relation and is defined by:

(Def. 19) $\mathrm{flow}_e(N) = (\text{the entrance of } N)^\smile \cup \text{the escape of } N \cup \mathrm{id}_{\text{the carrier of } N}$.

Next we state several propositions:

(29) $\mathrm{prox}_e(N) \subseteq [:\mathrm{shore}_e(N), \mathrm{shore}_e(N):]$ and $\mathrm{flow}_e(N) \subseteq [:\mathrm{shore}_e(N), \mathrm{shore}_e(N):]$.

(30) $\mathrm{prox}_e(N) \cdot \mathrm{prox}_e(N) = \mathrm{prox}_e(N)$ and $(\mathrm{prox}_e(N) \setminus \mathrm{id}_{\mathrm{shore}_e(N)}) \cdot \mathrm{prox}_e(N) = \emptyset$ and $\mathrm{prox}_e(N) \cup (\mathrm{prox}_e(N))^\smile \cup \mathrm{id}_{\mathrm{shore}_e(N)} = \mathrm{flow}_e(N) \cup (\mathrm{flow}_e(N))^\smile$.

(31)(i) $\mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the escape of } N)} \cdot ((\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) = (\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N}$, and

(ii) $\mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the entrance of } N)} \cdot ((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) = (\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N}$.

(32)(i) $((\text{The escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) \cdot ((\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$,

(ii) $((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) \cdot ((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$,

(iii) $((\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) \cdot ((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$, and

(iv) $((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) \cdot ((\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) = \emptyset$.

(33)(i) $((\text{The escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile \cdot ((\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile = \emptyset$, and

(ii) $((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile \cdot ((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile = \emptyset$.

(34)(i) $((\text{The escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile \cdot (\mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the escape of } N)})^\smile = ((\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile$, and

(ii) $((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile \cdot (\mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the entrance of } N)})^\smile = ((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile$.

(35)(i) $((\text{The escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) \cdot \mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the escape of } N)} = \emptyset$, and

(ii) $((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N}) \cdot \mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the entrance of } N)} = \emptyset$.

(36)(i) $\mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the escape of } N)} \cdot ((\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile = \emptyset$, and

(ii) $\mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the entrance of } N)} \cdot ((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile = \emptyset$.

Let us consider N. We introduce $\mathrm{support}_e(N)$ as a synonym of $\mathrm{shore}_e(N)$.
Let us consider N. The functor $\mathrm{entrance}_e(N)$ yields a binary relation and is defined by:

(Def. 21)[5] $\mathrm{entrance}_e(N) = ((\text{the escape of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile \cup \mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the escape of } N)}$.

The functor $\mathrm{escape}_e(N)$ yields a binary relation and is defined by:

(Def. 22) $\mathrm{escape}_e(N) = ((\text{the entrance of } N) \setminus \mathrm{id}_{\text{the carrier of } N})^\smile \cup \mathrm{id}_{(\text{the carrier of } N) \setminus \mathrm{rng}(\text{the entrance of } N)}$.

We now state two propositions:

(37) $\mathrm{entrance}_e(N) \cdot \mathrm{entrance}_e(N) = \mathrm{entrance}_e(N)$ and $\mathrm{entrance}_e(N) \cdot \mathrm{escape}_e(N) = \mathrm{entrance}_e(N)$ and $\mathrm{escape}_e(N) \cdot \mathrm{entrance}_e(N) = \mathrm{escape}_e(N)$ and $\mathrm{escape}_e(N) \cdot \mathrm{escape}_e(N) = \mathrm{escape}_e(N)$.

[4] The propositions (26) and (27) have been removed.
[5] The definition (Def. 20) has been removed.

(38) $\mathrm{entrance}_e(N)\cdot(\mathrm{entrance}_e(N)\setminus \mathrm{id}_{\mathrm{support}_e(N)})=\emptyset$ and $\mathrm{escape}_e(N)\cdot(\mathrm{escape}_e(N)\setminus \mathrm{id}_{\mathrm{support}_e(N)})=\emptyset$.

Let us consider N. We introduce $\mathrm{stanchion}_e(N)$ as a synonym of $\mathrm{shore}_e(N)$.

Let us consider N. The functor $\mathrm{adjac}_e(N)$ yields a binary relation and is defined as follows:

(Def. 24)[6] $\mathrm{adjac}_e(N)=(((\text{the entrance of }N)\cup(\text{the escape of }N))\setminus \mathrm{id}_{\text{the carrier of }N})\cup \mathrm{id}_{(\text{the carrier of }N)\setminus \mathrm{rng}(\text{the entrance of }N)}$.

We introduce $\mathrm{circulation}_e(N)$ as a synonym of $\mathrm{flow}_e(N)$.

One can prove the following two propositions:

(39) $\mathrm{adjac}_e(N)\subseteq [:\mathrm{stanchion}_e(N),\mathrm{stanchion}_e(N):]$ and $\mathrm{circulation}_e(N)\subseteq [:\mathrm{stanchion}_e(N),\mathrm{stanchion}_e(N):]$.

(40) $\mathrm{adjac}_e(N)\cdot \mathrm{adjac}_e(N)=\mathrm{adjac}_e(N)$ and $(\mathrm{adjac}_e(N)\setminus \mathrm{id}_{\mathrm{stanchion}_e(N)})\cdot \mathrm{adjac}_e(N)=\emptyset$ and $\mathrm{adjac}_e(N)\cup(\mathrm{adjac}_e(N))^{\smile}\cup \mathrm{id}_{\mathrm{stanchion}_e(N)}=\mathrm{circulation}_e(N)\cup(\mathrm{circulation}_e(N))^{\smile}$.

Let N be an E-net. We introduce $\mathrm{transitions}_s(N)$ as a synonym of $\mathrm{Places}_e(N)$. We introduce $\mathrm{places}_s(N)$ as a synonym of $\mathrm{Transitions}_e(N)$. We introduce $\mathrm{carrier}_s(N)$ as a synonym of $\mathrm{shore}_e(N)$. We introduce $\mathrm{enter}_s(N)$ as a synonym of $\mathrm{entrance}_e(N)$. We introduce $\mathrm{exit}_s(N)$ as a synonym of $\mathrm{escape}_e(N)$. We introduce $\mathrm{prox}_s(N)$ as a synonym of $\mathrm{adjac}_e(N)$.

In the sequel N is an E-net.

One can prove the following proposition

(41) $((\text{The entrance of }N)\setminus \mathrm{id}_{\text{the carrier of }N})^{\smile}\subseteq [:\mathrm{Places}_e(N),\mathrm{Transitions}_e(N):]$ and $((\text{the escape of }N)\setminus \mathrm{id}_{\text{the carrier of }N})^{\smile}\subseteq [:\mathrm{Places}_e(N),\mathrm{Transitions}_e(N):]$.

Let N be a G-net structure. The functor $\mathrm{pre}_s(N)$ yielding a binary relation is defined as follows:

(Def. 25) $\mathrm{pre}_s(N)=((\text{the escape of }N)\setminus \mathrm{id}_{\text{the carrier of }N})^{\smile}$.

The functor $\mathrm{post}_s(N)$ yielding a binary relation is defined as follows:

(Def. 26) $\mathrm{post}_s(N)=((\text{the entrance of }N)\setminus \mathrm{id}_{\text{the carrier of }N})^{\smile}$.

The following proposition is true

(42) $\mathrm{post}_s(N)\subseteq [:\mathrm{transitions}_s(N),\mathrm{places}_s(N):]$ and $\mathrm{pre}_s(N)\subseteq [:\mathrm{transitions}_s(N),\mathrm{places}_s(N):]$.

References

[1] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/zfmisc_1.html`.

[2] Waldemar Korczyński. Some properties of binary relations. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/sysrel.html`.

[3] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[4] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

Received January 31, 1992

Published January 2, 2004

[6] The definition (Def. 23) has been removed.