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Similarity of Formulae

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Summary. The main objective of the paper is to define the concept of the similarity of formulas. We mean by similar formulas the two formulas that differs only in the names of bound variables. Some authors (compare [14]) call such formulas *congruent*. We use the word *similar* following [12], [11], [13]. The concept is unjustfully neglected in many logical handbooks. It is intuitively quite clear, however the exact definition is not simple. As far as we know, only W.A. Pogorzelski and T. Prucnal [13] define it in the precise way. We follow basically the Pogorzelski's definition (compare [12]). We define renumeration of bound variables and we say that two formulas are similar if after renumeration are equal. Therefore we need a rule of chosing bound variables independent of the original choice. Quite obvious solution is to use consecutively variables $x_{k+1}, x_{k+2}, \ldots$, where k is the maximal index of free variable occurring in the formula. Therefore after the renumeration we get the new formula in which different quantifiers bind different variables. It is the reason that the result of renumeration applied to a formula φ we call φ *with variables separated*.

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The articles [19], [6], [24], [22], [17], [20], [25], [26], [4], [5], [16], [3], [10], [23], [21], [2], [18], [15], [1], [9], [7], and [8] provide the notation and terminology for this paper.

The following propositions are true:

(1) For all sets x, y and for every function f holds $(f+\cdot(\{x\} \longmapsto y))^\circ\{x\} = \{y\}$.

(2) For all sets K, L and for all sets x, y and for every function f holds $(f+\cdot(L \longmapsto y))^\circ K \subseteq f^\circ K \cup \{y\}$.

(3) For all sets x, y and for every function g and for every set A holds $(g+\cdot(\{x\} \longmapsto y))^\circ(A \setminus \{x\}) = g^\circ(A \setminus \{x\})$.

(4) For all sets x, y and for every function g and for every set A such that $y \notin g^\circ(A \setminus \{x\})$ holds $(g+\cdot(\{x\} \longmapsto y))^\circ(A \setminus \{x\}) = (g+\cdot(\{x\} \longmapsto y))^\circ A \setminus \{y\}$.

For simplicity, we follow the rules: p, q, r, s are elements of CQC-WFF, x is an element of BoundVar, i, k, l, m, n are elements of $\mathbb{N}$, l_1 is a variables list of k, and P is a k-ary predicate symbol.

We now state several propositions:

(5) If p is atomic, then there exist k, P, l_1 such that $p = P[l_1]$.

(6) If p is negative, then there exists q such that $p = \neg q$.

(7) If p is conjunctive, then there exist q, r such that $p = q \wedge r$.

(8) If p is universal, then there exist x, q such that $p = \forall_x q$.

(9) For every finite sequence l holds $\operatorname{rng} l = \{l(i) : 1 \leq i \wedge i \leq \operatorname{len} l\}$.

In this article we present several logical schemes. The scheme *QC Func ExN* deals with a non empty set $\mathcal{A}$, an element $\mathcal{B}$ of $\mathcal{A}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, a binary functor $\mathcal{G}$ yielding an element of $\mathcal{A}$, a ternary functor $\mathcal{H}$ yielding an element of $\mathcal{A}$, and a binary functor $\mathcal{I}$ yielding an element of $\mathcal{A}$, and states that:

There exists a function F from WFF into $\mathcal{A}$ such that
(i) $F(\mathrm{VERUM}) = \mathcal{B}$, and
(ii) for every element p of WFF holds if p is atomic, then $F(p) = \mathcal{F}(p)$ and if p is negative, then $F(p) = \mathcal{G}(F(\mathrm{Arg}(p)), p)$ and if p is conjunctive, then $F(p) = \mathcal{H}(F(\mathrm{LeftArg}(p)), F(\mathrm{RightArg}(p)), p)$ and if p is universal, then $F(p) = \mathcal{I}(F(\mathrm{Scope}(p)), p)$

for all values of the parameters.

The scheme *CQCF2 Func Ex* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, an element $\mathcal{C}$ of $\mathcal{B}^{\mathcal{A}}$, a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{B}^{\mathcal{A}}$, a binary functor $\mathcal{G}$ yielding an element of $\mathcal{B}^{\mathcal{A}}$, a 4-ary functor $\mathcal{H}$ yielding an element of $\mathcal{B}^{\mathcal{A}}$, and a ternary functor $\mathcal{I}$ yielding an element of $\mathcal{B}^{\mathcal{A}}$, and states that:

There exists a function F from CQC-WFF into $\mathcal{B}^{\mathcal{A}}$ such that
(i) $F(\mathrm{VERUM}) = \mathcal{C}$,
(ii) for every k and for every variables list l of k and for every k-ary predicate symbol P holds $F(P[l]) = \mathcal{F}(k, P, l)$, and
(iii) for all r, s, x holds $F(\neg r) = \mathcal{G}(F(r), r)$ and $F(r \wedge s) = \mathcal{H}(F(r), F(s), r, s)$ and $F(\forall_x r) = \mathcal{I}(x, F(r), r)$

for all values of the parameters.

The scheme *CQCF2 FUniq* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, functions $\mathcal{C}$, $\mathcal{D}$ from CQC-WFF into $\mathcal{B}^{\mathcal{A}}$, a function $\mathcal{E}$ from $\mathcal{A}$ into $\mathcal{B}$, a ternary functor $\mathcal{F}$ yielding a function from $\mathcal{A}$ into $\mathcal{B}$, a binary functor $\mathcal{G}$ yielding a function from $\mathcal{A}$ into $\mathcal{B}$, a 4-ary functor $\mathcal{H}$ yielding a function from $\mathcal{A}$ into $\mathcal{B}$, and a ternary functor $\mathcal{I}$ yielding a function from $\mathcal{A}$ into $\mathcal{B}$, and states that:

$\mathcal{C} = \mathcal{D}$

provided the following conditions are met:

- $\mathcal{C}(\mathrm{VERUM}) = \mathcal{E}$,
- For all k, l_1, P holds $\mathcal{C}(P[l_1]) = \mathcal{F}(k, P, l_1)$,
- For all r, s, x holds $\mathcal{C}(\neg r) = \mathcal{G}(\mathcal{C}(r), r)$ and $\mathcal{C}(r \wedge s) = \mathcal{H}(\mathcal{C}(r), \mathcal{C}(s), r, s)$ and $\mathcal{C}(\forall_x r) = \mathcal{I}(x, \mathcal{C}(r), r)$,
- $\mathcal{D}(\mathrm{VERUM}) = \mathcal{E}$,
- For all k, l_1, P holds $\mathcal{D}(P[l_1]) = \mathcal{F}(k, P, l_1)$, and
- For all r, s, x holds $\mathcal{D}(\neg r) = \mathcal{G}(\mathcal{D}(r), r)$ and $\mathcal{D}(r \wedge s) = \mathcal{H}(\mathcal{D}(r), \mathcal{D}(s), r, s)$ and $\mathcal{D}(\forall_x r) = \mathcal{I}(x, \mathcal{D}(r), r)$.

We now state four propositions:

(10) p is a subformula of $\neg p$.

(11) p is a subformula of $p \wedge q$ and q is a subformula of $p \wedge q$.

(12) p is a subformula of $\forall_x p$.

(13) For every variables list l of k and for every i such that $1 \leq i$ and $i \leq \operatorname{len} l$ holds $l(i) \in \mathrm{BoundVar}$.

Let D be a non empty set and let f be a function from D into CQC-WFF. The functor $\mathrm{NEG}(f)$ yields an element of CQC-WFF^D and is defined by:

(Def. 1) For every element a of D and for every element p of CQC-WFF such that $p = f(a)$ holds $(\mathrm{NEG}(f))(a) = \neg p$.

In the sequel f, h denote elements of $\mathrm{BoundVar}^{\mathrm{BoundVar}}$ and K denotes a finite subset of BoundVar.

Let f, g be functions from $[:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]$ into CQC-WFF and let n be a natural number. The functor $\mathrm{CON}(f, g, n)$ yielding an element of $\text{CQC-WFF}^{[:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]}$ is defined as follows:

(Def. 2) For all k, h, p, q such that $p = f(\langle k, h \rangle)$ and $q = g(\langle k+n, h \rangle)$ holds $(\mathrm{CON}(f, g, n))(\langle k, h \rangle) = p \wedge q$.

Let f be a function from $[:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]$ into CQC-WFF and let x be a bound variable. The functor $\mathrm{UNIV}(x, f)$ yields an element of $\text{CQC-WFF}^{[:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]}$ and is defined as follows:

(Def. 3) For all k, h, p such that $p = f(\langle k+1, h{+\cdot}(\{x\} \longmapsto \mathrm{x}_k)\rangle)$ holds $(\mathrm{UNIV}(x, f))(\langle k, h\rangle) = \forall_{\mathrm{x}_k} p$.

Let us consider k, let l be a variables list of k, and let f be an element of $\mathrm{BoundVar}^{\mathrm{BoundVar}}$. Then $f \cdot l$ is a variables list of k.

Let us consider k, let P be a k-ary predicate symbol, and let l be a variables list of k. The functor $\mathrm{ATOM}(P, l)$ yielding an element of $\text{CQC-WFF}^{[:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]}$ is defined by:

(Def. 4) For all n, h holds $(\mathrm{ATOM}(P, l))(\langle n, h\rangle) = P[h \cdot l]$.

Let us consider p. The number of quantifiers in p yields an element of $\mathbb{N}$ and is defined by the condition (Def. 5).

(Def. 5) There exists a function F from CQC-WFF into $\mathbb{N}$ such that
- (i) the number of quantifiers in $p = F(p)$,
- (ii) $F(\mathrm{VERUM}) = 0$, and
- (iii) for all r, s, x, k and for every variables list l of k and for every k-ary predicate symbol P holds $F(P[l]) = 0$ and $F(\neg r) = F(r)$ and $F(r \wedge s) = F(r) + F(s)$ and $F(\forall_x r) = F(r) + 1$.

Let f be a function from CQC-WFF into $\text{CQC-WFF}^{[:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]}$ and let x be an element of CQC-WFF. Then $f(x)$ is an element of $\text{CQC-WFF}^{[:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]}$.

The function Renum from CQC-WFF into $\text{CQC-WFF}^{[:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]}$ is defined by the conditions (Def. 6).

(Def. 6)(i) $\mathrm{Renum}(\mathrm{VERUM}) = [:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:] \longmapsto \mathrm{VERUM}$,
- (ii) for every k and for every variables list l of k and for every k-ary predicate symbol P holds $\mathrm{Renum}(P[l]) = \mathrm{ATOM}(P, l)$, and
- (iii) for all r, s, x holds $\mathrm{Renum}(\neg r) = \mathrm{NEG}(\mathrm{Renum}(r))$ and $\mathrm{Renum}(r \wedge s) = \mathrm{CON}(\mathrm{Renum}(r), \mathrm{Renum}(s), \text{the number of quantifiers in } r)$ and $\mathrm{Renum}(\forall_x r) = \mathrm{UNIV}(x, \mathrm{Renum}(r))$.

Let us consider p, k, f. The functor $\mathrm{Renum}_{k,f}(p)$ yielding an element of CQC-WFF is defined by:

(Def. 7) $\mathrm{Renum}_{k,f}(p) = \mathrm{Renum}(p)(\langle k, f\rangle)$.

Next we state several propositions:

(14) The number of quantifiers in $\mathrm{VERUM} = 0$.

(15) The number of quantifiers in $P[l_1] = 0$.

(16) The number of quantifiers in $\neg p =$ the number of quantifiers in p.

(17) The number of quantifiers in $p \wedge q =$ (the number of quantifiers in p) + (the number of quantifiers in q).

(18) The number of quantifiers in $\forall_x p =$ (the number of quantifiers in p) $+ 1$.

Let A be a non empty subset of $\mathbb{N}$. The functor $\min A$ yields a natural number and is defined as follows:

(Def. 8) $\min A \in A$ and for every k such that $k \in A$ holds $\min A \leq k$.

Next we state two propositions:

(19) For all non empty subsets A, B of $\mathbb{N}$ such that $A \subseteq B$ holds $\min B \leq \min A$.

(20) For every element p of WFF holds $\mathrm{snb}(p)$ is finite.

The scheme *MaxFinDomElem* deals with a non empty set $\mathcal{A}$, a set $\mathcal{B}$, and a binary predicate $\mathcal{P}$, and states that:

There exists an element x of $\mathcal{A}$ such that $x \in \mathcal{B}$ and for every element y of $\mathcal{A}$ such that $y \in \mathcal{B}$ holds $\mathcal{P}[x,y]$

provided the parameters meet the following conditions:

- $\mathcal{B}$ is finite and $\mathcal{B} \neq \emptyset$ and $\mathcal{B} \subseteq \mathcal{A}$,
- For all elements x, y of $\mathcal{A}$ holds $\mathcal{P}[x,y]$ or $\mathcal{P}[y,x]$, and
- For all elements x, y, z of $\mathcal{A}$ such that $\mathcal{P}[x,y]$ and $\mathcal{P}[y,z]$ holds $\mathcal{P}[x,z]$.

Let us consider p. The functor $\mathrm{NBI}(p)$ yielding a subset of $\mathbb{N}$ is defined as follows:

(Def. 9) $\mathrm{NBI}(p) = \{k : \bigwedge_i (k \leq i \Rightarrow \mathrm{x}_i \notin \mathrm{snb}(p))\}$.

Let us consider p. Note that $\mathrm{NBI}(p)$ is non empty.

Let us consider p. The functor $|\bullet : p|_{\mathbb{N}}$ yielding a natural number is defined by:

(Def. 10) $|\bullet : p|_{\mathbb{N}} = \min \mathrm{NBI}(p)$.

One can prove the following propositions:

(21) $|\bullet : p|_{\mathbb{N}} = 0$ iff p is closed.

(22) If $\mathrm{x}_i \in \mathrm{snb}(p)$, then $i < |\bullet : p|_{\mathbb{N}}$.

(23) $|\bullet : \mathrm{VERUM}|_{\mathbb{N}} = 0$.

(24) $|\bullet : \neg p|_{\mathbb{N}} = |\bullet : p|_{\mathbb{N}}$.

(25) $|\bullet : p|_{\mathbb{N}} \leq |\bullet : p \wedge q|_{\mathbb{N}}$ and $|\bullet : q|_{\mathbb{N}} \leq |\bullet : p \wedge q|_{\mathbb{N}}$.

Let C be a non empty set and let D be a non empty subset of C. Then id_D is an element of D^D.

Let us consider p. The functor p with variables separated yielding an element of CQC-WFF is defined by:

(Def. 11) p with variables separated $= \mathrm{Renum}_{|\bullet:p|_{\mathbb{N}}, \mathrm{id}_{\mathrm{BoundVar}}}(p)$.

One can prove the following proposition

(26) VERUM with variables separated = VERUM.

The scheme *CQCInd* concerns a unary predicate $\mathcal{P}$, and states that:

For every r holds $\mathcal{P}[r]$

provided the parameters meet the following requirements:

- $\mathcal{P}[\mathrm{VERUM}]$,
- For every k and for every variables list l of k and for every k-ary predicate symbol P holds $\mathcal{P}[P[l]]$,
- For every r such that $\mathcal{P}[r]$ holds $\mathcal{P}[\neg r]$,
- For all r, s such that $\mathcal{P}[r]$ and $\mathcal{P}[s]$ holds $\mathcal{P}[r \wedge s]$, and
- For all r, x such that $\mathcal{P}[r]$ holds $\mathcal{P}[\forall_x r]$.

Next we state four propositions:

(27) $P[l_1]$ with variables separated $= P[l_1]$.

(28) If p is atomic, then p with variables separated $= p$.

(29) $(\neg p)$ with variables separated $= \neg(p$ with variables separated$)$.

(30) If p is negative and $q = \mathrm{Arg}(p)$, then p with variables separated $= \neg(q$ with variables separated$)$.

Let us consider p and let X be a subset of $[:\mathrm{CQC\text{-}WFF}, \mathbb{N}, \mathrm{Fin\,BoundVar}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]$. We say that X is closed w.r.t. p if and only if the conditions (Def. 12) are satisfied.

(Def. 12)(i) $\langle p, |\bullet : p|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}\rangle \in X$,

(ii) for all q, k, K, f such that $\langle \neg q, k, K, f\rangle \in X$ holds $\langle q, k, K, f\rangle \in X$,

(iii) for all q, r, k, K, f such that $\langle q \wedge r, k, K, f\rangle \in X$ holds $\langle q, k, K, f\rangle \in X$ and $\langle r, k + \text{the number of quantifiers in } q, K, f\rangle \in X$, and

(iv) for all q, x, k, K, f such that $\langle \forall_x q, k, K, f\rangle \in X$ holds $\langle q, k+1, K \cup \{x\}, f +\cdot (\{x\} \longmapsto \mathrm{x}_k)\rangle \in X$.

Let D be a non empty set and let x be an element of D. Then $\{x\}$ is an element of $\mathrm{Fin}\, D$.

Let us consider p. The functor $\mathbf{Quadruples}_p$ yields a subset of $[:\mathrm{CQC\text{-}WFF}, \mathbb{N}, \mathrm{Fin\,BoundVar}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]$ and is defined by:

(Def. 13) $\mathbf{Quadruples}_p$ is closed w.r.t. p and for every subset D of $[:\mathrm{CQC\text{-}WFF}, \mathbb{N}, \mathrm{Fin\,BoundVar}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]$ such that D is closed w.r.t. p holds $\mathbf{Quadruples}_p \subseteq D$.

We now state several propositions:

(31) $\langle p, |\bullet : p|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}\rangle \in \mathbf{Quadruples}_p$.

(32) For all q, k, K, f such that $\langle \neg q, k, K, f\rangle \in \mathbf{Quadruples}_p$ holds $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_p$.

(33) For all q, r, k, K, f such that $\langle q \wedge r, k, K, f\rangle \in \mathbf{Quadruples}_p$ holds $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_p$ and $\langle r, k + \text{the number of quantifiers in } q, K, f\rangle \in \mathbf{Quadruples}_p$.

(34) For all q, x, k, K, f such that $\langle \forall_x q, k, K, f\rangle \in \mathbf{Quadruples}_p$ holds $\langle q, k+1, K \cup \{x\}, f +\cdot (\{x\} \longmapsto \mathrm{x}_k)\rangle \in \mathbf{Quadruples}_p$.

(35) Suppose $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_p$. Then

(i) $\langle q, k, K, f\rangle = \langle p, |\bullet : p|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}\rangle$, or

(ii) $\langle \neg q, k, K, f\rangle \in \mathbf{Quadruples}_p$, or

(iii) there exists r such that $\langle q \wedge r, k, K, f\rangle \in \mathbf{Quadruples}_p$, or

(iv) there exist r, l such that $k = l + \text{the number of quantifiers in } r$ and $\langle r \wedge q, l, K, f\rangle \in \mathbf{Quadruples}_p$, or

(v) there exist x, l, h such that $l + 1 = k$ but $h +\cdot (\{x\} \longmapsto \mathrm{x}_l) = f$ but $\langle \forall_x q, l, K, h\rangle \in \mathbf{Quadruples}_p$ or $\langle \forall_x q, l, K \setminus \{x\}, h\rangle \in \mathbf{Quadruples}_p$.

The scheme *Sep regression* deals with an element $\mathcal{A}$ of CQC-WFF and a 4-ary predicate $\mathcal{P}$, and states that:

For all q, k, K, f such that $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_{\mathcal{A}}$ holds $\mathcal{P}[q, k, K, f]$

provided the parameters meet the following conditions:

- $\mathcal{P}[\mathcal{A}, |\bullet : \mathcal{A}|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}]$,
- For all q, k, K, f such that $\langle \neg q, k, K, f\rangle \in \mathbf{Quadruples}_{\mathcal{A}}$ and $\mathcal{P}[\neg q, k, K, f]$ holds $\mathcal{P}[q, k, K, f]$,
- For all q, r, k, K, f such that $\langle q \wedge r, k, K, f\rangle \in \mathbf{Quadruples}_{\mathcal{A}}$ and $\mathcal{P}[q \wedge r, k, K, f]$ holds $\mathcal{P}[q, k, K, f]$ and $\mathcal{P}[r, k + \text{the number of quantifiers in } q, K, f]$, and
- For all q, x, k, K, f such that $\langle \forall_x q, k, K, f\rangle \in \mathbf{Quadruples}_{\mathcal{A}}$ and $\mathcal{P}[\forall_x q, k, K, f]$ holds $\mathcal{P}[q, k+1, K \cup \{x\}, f +\cdot (\{x\} \longmapsto \mathrm{x}_k)]$.

Next we state a number of propositions:

(36) For all q, k, K, f such that $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_p$ holds q is a subformula of p.

(37) $\mathbf{Quadruples}_{\mathrm{VERUM}} = \{\langle \mathrm{VERUM}, 0, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}\rangle\}$.

(38) For every k and for every variables list l of k and for every k-ary predicate symbol P holds $\mathbf{Quadruples}_{P[l]} = \{\langle P[l], |\bullet : P[l]|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}\rangle\}$.

(39) For all q, k, K, f such that $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_p$ holds $\mathrm{snb}(q) \subseteq \mathrm{snb}(p) \cup K$.

(40) If $\langle q,m,K,f\rangle \in \mathbf{Quadruples}_p$ and $x_i \in f^\circ K$, then $i < m$.

(41) If $\langle q,m,K,f\rangle \in \mathbf{Quadruples}_p$, then $x_m \notin f^\circ K$.

(42) If $\langle q,m,K,f\rangle \in \mathbf{Quadruples}_p$ and $x_i \in f^\circ \operatorname{snb}(p)$, then $i < m$.

(43) If $\langle q,m,K,f\rangle \in \mathbf{Quadruples}_p$ and $x_i \in f^\circ \operatorname{snb}(q)$, then $i < m$.

(44) If $\langle q,m,K,f\rangle \in \mathbf{Quadruples}_p$, then $x_m \notin f^\circ \operatorname{snb}(q)$.

(45) $\operatorname{snb}(p) = \operatorname{snb}(p\,\text{with variables separated})$.

(46) $|\bullet : p|_{\mathbb{N}} = |\bullet : p\,\text{with variables separated}|_{\mathbb{N}}$.

Let us consider p, q. We say that p and q are similar if and only if:

(Def. 14) $p\,\text{with variables separated} = q\,\text{with variables separated}$.

Let us notice that the predicate p and q are similar is reflexive and symmetric.

One can prove the following proposition

(49)[1] If p and q are similar and q and r are similar, then p and r are similar.

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