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Metric-Affine Configurations in Metric Affine Planes — Part II

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Summary. A continuation of [5]. We introduce more configurational axioms i.e. orthogonalizations of "scherungssatzes" (direct and indirect), "Scherungssatz" with orthogonal axes, Pappus axiom with orthogonal axes; we also consider the affine Major Pappus Axiom and affine minor Desargues Axiom. We prove a number of implications which hold between the above axioms.

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The articles [6], [2], [4], [1], [3], and [5] provide the notation and terminology for this paper.

We follow the rules: X is a metric affine plane, o, a, a_1, a_2, a_3, a_4, b, b_1, b_2, b_3, b_4, c, c_1, d are elements of X, and A, K, M, N are subsets of X.

Let us consider X. We say that X satisfies Pappos Axiom with orthogonal axes if and only if the condition (Def. 1) is satisfied.

(Def. 1) Let given o, a_1, a_2, a_3, b_1, b_2, b_3, M, N. Suppose that $o \in M$ and $a_1 \in M$ and $a_2 \in M$ and $a_3 \in M$ and $o \in N$ and $b_1 \in N$ and $b_2 \in N$ and $b_3 \in N$ and $b_2 \notin M$ and $a_3 \notin N$ and $M \perp N$ and $o \neq a_1$ and $o \neq a_2$ and $o \neq a_3$ and $o \neq b_1$ and $o \neq b_2$ and $o \neq b_3$ and $a_3, b_2 \upuparrows a_2, b_1$ and $a_3, b_3 \upuparrows a_1, b_1$. Then $a_1, b_2 \upuparrows a_2, b_3$.

We introduce Pappos Axiom with orthogonal axes holds in X as a synonym of X satisfies Pappos Axiom with orthogonal axes. We say that X satisfies Pappos Axiom if and only if the condition (Def. 2) is satisfied.

(Def. 2) Let given o, a_1, a_2, a_3, b_1, b_2, b_3, M, N. Suppose that M is a line and N is a line and $o \in M$ and $a_1 \in M$ and $a_2 \in M$ and $a_3 \in M$ and $o \in N$ and $b_1 \in N$ and $b_2 \in N$ and $b_3 \in N$ and $b_2 \notin M$ and $a_3 \notin N$ and $o \neq a_1$ and $o \neq a_2$ and $o \neq a_3$ and $o \neq b_1$ and $o \neq b_2$ and $o \neq b_3$ and $a_3, b_2 \upuparrows a_2, b_1$ and $a_3, b_3 \upuparrows a_1, b_1$. Then $a_1, b_2 \upuparrows a_2, b_3$.

We introduce Pappos Axiom holds in X as a synonym of X satisfies Pappos Axiom. We say that X satisfies MH1 if and only if the condition (Def. 3) is satisfied.

(Def. 3) Let given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that $M \perp N$ and $a_1 \in M$ and $a_3 \in M$ and $b_1 \in M$ and $b_3 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_2 \in N$ and $b_4 \in N$ and $a_2 \notin M$ and $a_4 \notin M$ and $a_1, a_2 \perp b_1, b_2$ and $a_2, a_3 \perp b_2, b_3$ and $a_3, a_4 \perp b_3, b_4$. Then $a_1, a_4 \perp b_1, b_4$.

We introduce MH1 holds in X as a synonym of X satisfies MH1. We say that X satisfies MH2 if and only if the condition (Def. 4) is satisfied.

(Def. 4) Let given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that $M \perp N$ and $a_1 \in M$ and $a_3 \in M$ and $b_2 \in M$ and $b_4 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_1 \in N$ and $b_3 \in N$ and $a_2 \notin M$ and $a_4 \notin M$ and $a_1,a_2 \perp b_1,b_2$ and $a_2,a_3 \perp b_2,b_3$ and $a_3,a_4 \perp b_3,b_4$. Then $a_1,a_4 \perp b_1,b_4$.

We introduce MH2 holds in X as a synonym of X satisfies MH2. We say that X satisfies trapezium variant of Desargues Axiom if and only if the condition (Def. 5) is satisfied.

(Def. 5) Let given o, a, a_1, b, b_1, c, c_1. Suppose that $o \neq a$ and $o \neq a_1$ and $o \neq b$ and $o \neq b_1$ and $o \neq c$ and $o \neq c_1$ and not $\mathbf{L}(b,b_1,a)$ and not $\mathbf{L}(b,b_1,c)$ and $\mathbf{L}(o,a,a_1)$ and $\mathbf{L}(o,b,b_1)$ and $\mathbf{L}(o,c,c_1)$ and $a,b \upuparrows a_1,b_1$ and $a,b \upuparrows o,c$ and $b,c \upuparrows b_1,c_1$. Then $a,c \upuparrows a_1,c_1$.

We introduce trapezium variant of Desargues Axiom holds in X as a synonym of X satisfies trapezium variant of Desargues Axiom. We say that X satisfies Scherungssatz if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that M is a line and N is a line and $a_1 \in M$ and $a_3 \in M$ and $b_1 \in M$ and $b_3 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_2 \in N$ and $b_4 \in N$ and $a_4 \notin M$ and $a_2 \notin M$ and $b_2 \notin M$ and $b_4 \notin M$ and $a_1 \notin N$ and $a_3 \notin N$ and $b_1 \notin N$ and $b_3 \notin N$ and $a_3,a_2 \upuparrows b_3,b_2$ and $a_2,a_1 \upuparrows b_2,b_1$ and $a_1,a_4 \upuparrows b_1,b_4$. Then $a_3,a_4 \upuparrows b_3,b_4$.

We introduce Scherungssatz holds in X as a synonym of X satisfies Scherungssatz. We say that X satisfies Scherungssatz with orthogonal axes if and only if the condition (Def. 7) is satisfied.

(Def. 7) Let given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that $M \perp N$ and $a_1 \in M$ and $a_3 \in M$ and $b_1 \in M$ and $b_3 \in M$ and $a_2 \in N$ and $a_4 \in N$ and $b_2 \in N$ and $b_4 \in N$ and $a_4 \notin M$ and $a_2 \notin M$ and $b_2 \notin M$ and $b_4 \notin M$ and $a_1 \notin N$ and $a_3 \notin N$ and $b_1 \notin N$ and $b_3 \notin N$ and $a_3,a_2 \upuparrows b_3,b_2$ and $a_2,a_1 \upuparrows b_2,b_1$ and $a_1,a_4 \upuparrows b_1,b_4$. Then $a_3,a_4 \upuparrows b_3,b_4$.

We introduce Scherungssatz with orthogonal axes holds in X as a synonym of X satisfies Scherungssatz with orthogonal axes. We say that X satisfies des if and only if:

(Def. 8) For all a, a_1, b, b_1, c, c_1 such that not $\mathbf{L}(a,a_1,b)$ and not $\mathbf{L}(a,a_1,c)$ and $a,a_1 \upuparrows b,b_1$ and $a,a_1 \upuparrows c,c_1$ and $a,b \upuparrows a_1,b_1$ and $a,c \upuparrows a_1,c_1$ holds $b,c \upuparrows b_1,c_1$.

We introduce minor Desargues Axiom holds in X as a synonym of X satisfies des.

The following propositions are true:

(1) There exist a, b, c such that $\mathbf{L}(a,b,c)$ and $a \neq b$ and $b \neq c$ and $c \neq a$.

(2) For all a, b such that $a \neq b$ there exists c such that $\mathbf{L}(a,b,c)$ and $a \neq c$ and $b \neq c$.

(3) For all A, a such that A is a line there exists K such that $a \in K$ and $A \perp K$.

(4) If A is a line and $a \in A$ and $b \in A$ and $c \in A$, then $\mathbf{L}(a,b,c)$.

(5) If A is a line and M is a line and $a \in A$ and $b \in A$ and $a \in M$ and $b \in M$, then $a = b$ or $A = M$.

(6) Let given a, b, c, d, M, M' be a subset of the affine reduct of X, and c', d' be elements of the affine reduct of X. If $c = c'$ and $d = d'$ and $M = M'$ and $a \in M$ and $b \in M$ and c', d' // M', then $c,d \upuparrows a,b$.

(7) Suppose trapezium variant of Desargues Axiom holds in X. Then the affine reduct of X satisfies **TDES**.

(8) If the affine reduct of X satisfies **des**, then minor Desargues Axiom holds in X.

(9) If MH1 holds in X, then Scherungssatz with orthogonal axes holds in X.

(10) If MH2 holds in X, then Scherungssatz with orthogonal axes holds in X.

(11) If AH holds in X, then trapezium variant of Desargues Axiom holds in X.

(12) Suppose Scherungssatz with orthogonal axes holds in X and trapezium variant of Desargues Axiom holds in X. Then Scherungssatz holds in X.

(13) Suppose Pappos Axiom with orthogonal axes holds in X and Desargues Axiom holds in X. Then Pappos Axiom holds in X.

(14) If MH1 holds in X and MH2 holds in X, then Pappos Axiom with orthogonal axes holds in X.

(15) If theorem on three perpendiculars holds in X, then Pappos Axiom with orthogonal axes holds in X.

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