JOURNAL OF FORMALIZED MATHEMATICS
Volume 5, Released 1993, Published 2003
Inst. of Computer Science, Univ. of Białystok

Complex Sequences

Agnieszka Banachowicz
Warsaw University
Białystok

Anna Winnicka
Warsaw University
Białystok

Summary. Definitions of complex sequence and operations on sequences (multiplication of sequences and multiplication by a complex number, addition, subtraction, division and absolute value of sequence) are given. We followed [4].

MML Identifier: COMSEQ_1.

WWW: http://mizar.org/JFM/Vol5/comseq_1.html

The articles [5], [8], [6], [3], [9], [1], [10], [2], [7], and [4] provide the notation and terminology for this paper.

For simplicity, we use the following convention: f is a function, n is a natural number, r, p are elements of $\mathbb{C}$, and x is a set.

A complex sequence is a function from $\mathbb{N}$ into $\mathbb{C}$.

In the sequel s_1, s_2, s_3, s_4, s'_1, s'_2 denote complex sequences.

One can prove the following two propositions:

(1) f is a complex sequence iff $\operatorname{dom} f = \mathbb{N}$ and for every x such that $x \in \mathbb{N}$ holds $f(x)$ is an element of $\mathbb{C}$.

(2) f is a complex sequence iff $\operatorname{dom} f = \mathbb{N}$ and for every n holds $f(n)$ is an element of $\mathbb{C}$.

Let us consider s_1, n. Then $s_1(n)$ is an element of $\mathbb{C}$.

The scheme *ExComplexSeq* deals with a unary functor $\mathcal{F}$ yielding an element of $\mathbb{C}$, and states that:

There exists s_1 such that for every n holds $s_1(n) = \mathcal{F}(n)$

for all values of the parameter.

Let I_1 be a complex sequence. We say that I_1 is non-zero if and only if:

(Def. 1) $\operatorname{rng} I_1 \subseteq \mathbb{C} \setminus \{0_{\mathbb{C}}\}$.

We now state the proposition

(3) s_1 is non-zero iff for every x such that $x \in \mathbb{N}$ holds $s_1(x) \neq 0_{\mathbb{C}}$.

Let us observe that there exists a complex sequence which is non-zero.

The following propositions are true:

(4) s_1 is non-zero iff for every n holds $s_1(n) \neq 0_{\mathbb{C}}$.

(6)[1] For all s_1, s_2 such that for every n holds $s_1(n) = s_2(n)$ holds $s_1 = s_2$.

(7) For every r there exists s_1 such that $\operatorname{rng} s_1 = \{r\}$.

[1] The proposition (5) has been removed.

Let C be a non empty set and let f_1, f_2 be partial functions from C to $\mathbb{C}$. The functor $f_1 + f_2$ yielding a partial function from C to $\mathbb{C}$ is defined by:

(Def. 2) $\operatorname{dom}(f_1 + f_2) = \operatorname{dom} f_1 \cap \operatorname{dom} f_2$ and for every element c of C such that $c \in \operatorname{dom}(f_1 + f_2)$ holds $(f_1 + f_2)(c) = (f_1)_c + (f_2)_c$.

Let us note that the functor $f_1 + f_2$ is commutative. The functor $f_1\ f_2$ yields a partial function from C to $\mathbb{C}$ and is defined by:

(Def. 3) $\operatorname{dom}(f_1\ f_2) = \operatorname{dom} f_1 \cap \operatorname{dom} f_2$ and for every element c of C such that $c \in \operatorname{dom}(f_1\ f_2)$ holds $(f_1\ f_2)(c) = (f_1)_c \cdot (f_2)_c$.

Let us observe that the functor $f_1\ f_2$ is commutative.

Let C be a non empty set and let f_1, f_2 be functions from C into $\mathbb{C}$. Then $f_1 + f_2$ can be characterized by the condition:

(Def. 4) $\operatorname{dom}(f_1 + f_2) = C$ and for every element c of C holds $(f_1 + f_2)(c) = f_1(c) + f_2(c)$.

Then $f_1\ f_2$ can be characterized by the condition:

(Def. 5) $\operatorname{dom}(f_1\ f_2) = C$ and for every element c of C holds $(f_1\ f_2)(c) = f_1(c) \cdot f_2(c)$.

Let C be a non empty set and let s_2, s_3 be functions from C into $\mathbb{C}$. Note that $s_2 + s_3$ is total and $s_2\ s_3$ is total.

Let C be a non empty set, let f be a partial function from C to $\mathbb{C}$, and let us consider r. The functor $r\ f$ yielding a partial function from C to $\mathbb{C}$ is defined by:

(Def. 6) $\operatorname{dom}(r\ f) = \operatorname{dom} f$ and for every element c of C such that $c \in \operatorname{dom}(r\ f)$ holds $(r\ f)(c) = r \cdot f_c$.

Let C be a non empty set, let f be a function from C into $\mathbb{C}$, and let us consider r. Then $r\ f$ can be characterized by the condition:

(Def. 7) $\operatorname{dom}(r\ f) = C$ and for every element n of C holds $(r\ f)(n) = r \cdot f(n)$.

Let C be a non empty set, let s_1 be a function from C into $\mathbb{C}$, and let us consider r. Observe that $r\ s_1$ is total.

Let C be a non empty set and let f be a partial function from C to $\mathbb{C}$. The functor $-f$ yielding a partial function from C to $\mathbb{C}$ is defined as follows:

(Def. 8) $\operatorname{dom}(-f) = \operatorname{dom} f$ and for every element c of C such that $c \in \operatorname{dom}(-f)$ holds $(-f)(c) = -f_c$.

Let C be a non empty set and let f be a function from C into $\mathbb{C}$. Then $-f$ can be characterized by the condition:

(Def. 9) $\operatorname{dom}(-f) = C$ and for every element n of C holds $(-f)(n) = -f(n)$.

Let C be a non empty set and let s_1 be a function from C into $\mathbb{C}$. Observe that $-s_1$ is total.

Let C be a non empty set and let f_1, f_2 be partial functions from C to $\mathbb{C}$. The functor $f_1 - f_2$ yielding a partial function from C to $\mathbb{C}$ is defined by:

(Def. 10) $f_1 - f_2 = f_1 + -f_2$.

Let C be a non empty set and let f_1, f_2 be functions from C into $\mathbb{C}$. Observe that $f_1 - f_2$ is total.

Let us consider s_1. The functor ${s_1}^{-1}$ yielding a complex sequence is defined by:

(Def. 11) For every n holds ${s_1}^{-1}(n) = s_1(n)^{-1}$.

Let us consider s_2, s_1. The functor s_2/s_1 yielding a complex sequence is defined by:

(Def. 12) $s_2/s_1 = s_2\ {s_1}^{-1}$.

Let C be a non empty set and let f be a partial function from C to $\mathbb{C}$. The functor $|f|$ yielding a partial function from C to $\mathbb{R}$ is defined by:

(Def. 13) $\operatorname{dom}|f| = \operatorname{dom} f$ and for every element c of C such that $c \in \operatorname{dom}|f|$ holds $|f|(c) = |f_c|$.

Let C be a non empty set and let s_1 be a function from C into $\mathbb{C}$. Then $|s_1|$ can be characterized by the condition:

(Def. 14) $\operatorname{dom}|s_1| = C$ and for every element n of C holds $|s_1|(n) = |s_1(n)|$.

Let C be a non empty set and let s_1 be a function from C into $\mathbb{C}$. Observe that $|s_1|$ is total.

We now state a number of propositions:

(9)[2] $(s_2 + s_3) + s_4 = s_2 + (s_3 + s_4)$.

(11)[3] $(s_2\, s_3)\, s_4 = s_2\, (s_3\, s_4)$.

(12) $(s_2 + s_3)\, s_4 = s_2\, s_4 + s_3\, s_4$.

(13) $s_4\, (s_2 + s_3) = s_4\, s_2 + s_4\, s_3$.

(14) $-s_1 = (-1_{\mathbb{C}})\, s_1$.

(15) $r\, (s_2\, s_3) = (r\, s_2)\, s_3$.

(16) $r\, (s_2\, s_3) = s_2\, (r\, s_3)$.

(17) $(s_2 - s_3)\, s_4 = s_2\, s_4 - s_3\, s_4$.

(18) $s_4\, s_2 - s_4\, s_3 = s_4\, (s_2 - s_3)$.

(19) $r\, (s_2 + s_3) = r\, s_2 + r\, s_3$.

(20) $(r \cdot p)\, s_1 = r\, (p\, s_1)$.

(21) $r\, (s_2 - s_3) = r\, s_2 - r\, s_3$.

(22) $r\, (s_2 / s_1) = (r\, s_2) / s_1$.

(23) $s_2 - (s_3 + s_4) = s_2 - s_3 - s_4$.

(24) $1_{\mathbb{C}}\, s_1 = s_1$.

(25) $--s_1 = s_1$.

(26) $s_2 - -s_3 = s_2 + s_3$.

(27) $s_2 - (s_3 - s_4) = (s_2 - s_3) + s_4$.

(28) $s_2 + (s_3 - s_4) = (s_2 + s_3) - s_4$.

(29) $(-s_2)\, s_3 = -s_2\, s_3$ and $s_2\, -s_3 = -s_2\, s_3$.

(30) If s_1 is non-zero, then $s_1{}^{-1}$ is non-zero.

(31) $(s_1{}^{-1})^{-1} = s_1$.

(32) s_1 is non-zero and s_2 is non-zero iff $s_1\, s_2$ is non-zero.

(33) If s_1 is non-zero and s_2 is non-zero, then $s_1{}^{-1}\, s_2{}^{-1} = (s_1\, s_2)^{-1}$.

(34) If s_1 is non-zero, then $(s_2 / s_1)\, s_1 = s_2$.

(35) If s_1 is non-zero and s_2 is non-zero, then $(s'_1 / s_1)\, (s'_2 / s_2) = (s'_1\, s'_2) / (s_1\, s_2)$.

(36) If s_1 is non-zero and s_2 is non-zero, then s_1 / s_2 is non-zero.

[2] The proposition (8) has been removed.

[3] The proposition (10) has been removed.

(37) If s_1 is non-zero and s_2 is non-zero, then $(s_1/s_2)^{-1} = s_2/s_1$.

(38) $s_3\,(s_2/s_1) = (s_3\,s_2)/s_1$.

(39) If s_1 is non-zero and s_2 is non-zero, then $s_3/(s_1/s_2) = (s_3\,s_2)/s_1$.

(40) If s_1 is non-zero and s_2 is non-zero, then $s_3/s_1 = (s_3\,s_2)/(s_1\,s_2)$.

(41) If $r \neq 0_{\mathbb{C}}$ and s_1 is non-zero, then $r\,s_1$ is non-zero.

(42) If s_1 is non-zero, then $-s_1$ is non-zero.

(43) If $r \neq 0_{\mathbb{C}}$ and s_1 is non-zero, then $(r\,s_1)^{-1} = r^{-1}\,{s_1}^{-1}$.

(44) If s_1 is non-zero, then $(-s_1)^{-1} = (-1_{\mathbb{C}})\,{s_1}^{-1}$.

(45) If s_1 is non-zero, then $-s_2/s_1 = (-s_2)/s_1$ and $s_2/-s_1 = -s_2/s_1$.

(46) $s_2/s_1 + s'_2/s_1 = (s_2 + s'_2)/s_1$ and $s_2/s_1 - s'_2/s_1 = (s_2 - s'_2)/s_1$.

(47) If s_1 is non-zero and s'_1 is non-zero, then $s_2/s_1 + s'_2/s'_1 = (s_2\,s'_1 + s'_2\,s_1)/(s_1\,s'_1)$ and $s_2/s_1 - s'_2/s'_1 = (s_2\,s'_1 - s'_2\,s_1)/(s_1\,s'_1)$.

(48) If s_1 is non-zero and s'_1 is non-zero and s_2 is non-zero, then $s'_2/s_1/(s'_1/s_2) = (s'_2\,s_2)/(s_1\,s'_1)$.

(49) $|s_1\,s'_1| = |s_1|\,|s'_1|$.

(50) If s_1 is non-zero, then $|s_1|$ is non-zero.

(51) If s_1 is non-zero, then $|s_1|^{-1} = |{s_1}^{-1}|$.

(52) If s_1 is non-zero, then $|s'_1/s_1| = |s'_1|/|s_1|$.

(53) $|r\,s_1| = |r|\,|s_1|$.

REFERENCES

[1] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[2] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_2.html`.

[3] Czesław Byliński. The complex numbers. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/complex1.html`.

[4] Jarosław Kotowicz. Real sequences and basic operations on them. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/seq_1.html`.

[5] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[6] Andrzej Trybulec. Subsets of real numbers. *Journal of Formalized Mathematics*, Addenda, 2003. `http://mizar.org/JFM/Addenda/numbers.html`.

[7] Wojciech A. Trybulec. Pigeon hole principle. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/finseq_4.html`.

[8] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[9] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

[10] Edmund Woronowicz. Relations defined on sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relset_1.html.

Received November 5, 1993

Published January 2, 2004