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Products and Coproducts in Categories

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Summary. A product and coproduct in categories are introduced. The concepts included correspond to that presented in [6].

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The articles [8], [10], [11], [1], [7], [4], [2], [9], [3], and [5] provide the notation and terminology for this paper.

1. Indexed families

For simplicity, we adopt the following rules: I is a set, x, x_1, x_2, y are sets, A is a non empty set, C, D are categories, a, b, c, d are objects of C, and f, g, h, k, p_1, p_2, q_1, q_2, i_1, i_2, j_1, j_2 are morphisms of C.

Let us consider I, A, let F be a function from I into A, and let x be a set. Let us assume that $x \in I$. Then F_x can be characterized by the condition:

(Def. 1) $F_x = F(x)$.

The scheme *LambdaIdx* deals with a set $\mathcal{A}$, a non empty set $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and states that:

There exists a function F from $\mathcal{A}$ into $\mathcal{B}$ such that for every x such that $x \in \mathcal{A}$ holds $F_x = \mathcal{F}(x)$

for all values of the parameters.

We now state the proposition

(1) For all functions F_1, F_2 from I into A such that for every x such that $x \in I$ holds $(F_1)_x = (F_2)_x$ holds $F_1 = F_2$.

The scheme *FuncIdx correctness* deals with a set $\mathcal{A}$, a non empty set $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and states that:

(i) There exists a function F from $\mathcal{A}$ into $\mathcal{B}$ such that for every x such that $x \in \mathcal{A}$ holds $F_x = \mathcal{F}(x)$, and

(ii) for all functions F_1, F_2 from $\mathcal{A}$ into $\mathcal{B}$ such that for every x such that $x \in \mathcal{A}$ holds $(F_1)_x = \mathcal{F}(x)$ and for every x such that $x \in \mathcal{A}$ holds $(F_2)_x = \mathcal{F}(x)$ holds $F_1 = F_2$

for all values of the parameters.

Let us consider A, I and let a be an element of A. Then $I \longmapsto a$ is a function from I into A.

One can prove the following two propositions:

(2) For every element a of A such that $x \in I$ holds $(I \longmapsto a)_x = a$.

(7)[1] If $x_1 \neq x_2$, then for all elements y_1, y_2 of A holds $[x_1 \longmapsto y_1, x_2 \longmapsto y_2]_{x_1} = y_1$ and $[x_1 \longmapsto y_1, x_2 \longmapsto y_2]_{x_2} = y_2$.

2. Indexed families of morphisms

Let us consider C, I and let F be a function from I into the morphisms of C. The functor $\mathrm{dom}_\kappa F(\kappa)$ yields a function from I into the objects of C and is defined by:

(Def. 3)[2] For every x such that $x \in I$ holds $(\mathrm{dom}_\kappa F(\kappa))_x = \mathrm{dom}(F_x)$.

The functor $\mathrm{cod}_\kappa F(\kappa)$ yielding a function from I into the objects of C is defined by:

(Def. 4) For every x such that $x \in I$ holds $(\mathrm{cod}_\kappa F(\kappa))_x = \mathrm{cod}(F_x)$.

Next we state four propositions:

(8) $\mathrm{dom}_\kappa(I \longmapsto f)(\kappa) = I \longmapsto \mathrm{dom}\, f$.

(9) $\mathrm{cod}_\kappa(I \longmapsto f)(\kappa) = I \longmapsto \mathrm{cod}\, f$.

(10) $\mathrm{dom}_\kappa[x_1 \longmapsto p_1, x_2 \longmapsto p_2](\kappa) = [x_1 \longmapsto \mathrm{dom}\, p_1, x_2 \longmapsto \mathrm{dom}\, p_2]$.

(11) $\mathrm{cod}_\kappa[x_1 \longmapsto p_1, x_2 \longmapsto p_2](\kappa) = [x_1 \longmapsto \mathrm{cod}\, p_1, x_2 \longmapsto \mathrm{cod}\, p_2]$.

Let us consider C, I and let F be a function from I into the morphisms of C. The functor F^{op} yielding a function from I into the morphisms of C^{op} is defined by:

(Def. 5) For every x such that $x \in I$ holds $(F^{\mathrm{op}})_x = (F_x)^{\mathrm{op}}$.

One can prove the following three propositions:

(12) $(I \longmapsto f)^{\mathrm{op}} = I \longmapsto f^{\mathrm{op}}$.

(13) If $x_1 \neq x_2$, then $[x_1 \longmapsto p_1, x_2 \longmapsto p_2]^{\mathrm{op}} = [x_1 \longmapsto {p_1}^{\mathrm{op}}, x_2 \longmapsto {p_2}^{\mathrm{op}}]$.

(14) For every function F from I into the morphisms of C holds $(F^{\mathrm{op}})^{\mathrm{op}} = F$.

Let us consider C, I and let F be a function from I into the morphisms of C^{op}. The functor ${}^{\mathrm{op}}F$ yields a function from I into the morphisms of C and is defined by:

(Def. 6) For every x such that $x \in I$ holds $({}^{\mathrm{op}}F)_x = {}^{\mathrm{op}}(F_x)$.

We now state three propositions:

(15) For every morphism f of C^{op} holds ${}^{\mathrm{op}}(I \longmapsto f) = I \longmapsto {}^{\mathrm{op}}f$.

(16) If $x_1 \neq x_2$, then for all morphisms p_1, p_2 of C^{op} holds ${}^{\mathrm{op}}[x_1 \longmapsto p_1, x_2 \longmapsto p_2] = [x_1 \longmapsto {}^{\mathrm{op}}p_1, x_2 \longmapsto {}^{\mathrm{op}}p_2]$.

(17) For every function F from I into the morphisms of C holds ${}^{\mathrm{op}}(F^{\mathrm{op}}) = F$.

Let us consider C, I, let F be a function from I into the morphisms of C, and let us consider f. The functor $F \cdot f$ yielding a function from I into the morphisms of C is defined by:

(Def. 7) For every x such that $x \in I$ holds $(F \cdot f)_x = F_x \cdot f$.

The functor $f \cdot F$ yielding a function from I into the morphisms of C is defined by:

(Def. 8) For every x such that $x \in I$ holds $(f \cdot F)_x = f \cdot F_x$.

We now state four propositions:

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[1] The propositions (3)–(6) have been removed.
[2] The definition (Def. 2) has been removed.

(18) If $x_1 \neq x_2$, then $[x_1 \longmapsto p_1, x_2 \longmapsto p_2] \cdot f = [x_1 \longmapsto p_1 \cdot f, x_2 \longmapsto p_2 \cdot f]$.

(19) If $x_1 \neq x_2$, then $f \cdot [x_1 \longmapsto p_1, x_2 \longmapsto p_2] = [x_1 \longmapsto f \cdot p_1, x_2 \longmapsto f \cdot p_2]$.

(20) For every function F from I into the morphisms of C such that $\mathrm{dom}_\kappa F(\kappa) = I \longmapsto \mathrm{cod}\, f$ holds $\mathrm{dom}_\kappa (F \cdot f)(\kappa) = I \longmapsto \mathrm{dom}\, f$ and $\mathrm{cod}_\kappa (F \cdot f)(\kappa) = \mathrm{cod}_\kappa F(\kappa)$.

(21) For every function F from I into the morphisms of C such that $\mathrm{cod}_\kappa F(\kappa) = I \longmapsto \mathrm{dom}\, f$ holds $\mathrm{dom}_\kappa (f \cdot F)(\kappa) = \mathrm{dom}_\kappa F(\kappa)$ and $\mathrm{cod}_\kappa (f \cdot F)(\kappa) = I \longmapsto \mathrm{cod}\, f$.

Let us consider C, I and let F, G be functions from I into the morphisms of C. The functor $F \cdot G$ yielding a function from I into the morphisms of C is defined as follows:

(Def. 9) For every x such that $x \in I$ holds $(F \cdot G)_x = F_x \cdot G_x$.

We now state four propositions:

(22) For all functions F, G from I into the morphisms of C such that $\mathrm{dom}_\kappa F(\kappa) = \mathrm{cod}_\kappa G(\kappa)$ holds $\mathrm{dom}_\kappa (F \cdot G)(\kappa) = \mathrm{dom}_\kappa G(\kappa)$ and $\mathrm{cod}_\kappa (F \cdot G)(\kappa) = \mathrm{cod}_\kappa F(\kappa)$.

(23) If $x_1 \neq x_2$, then $[x_1 \longmapsto p_1, x_2 \longmapsto p_2] \cdot [x_1 \longmapsto q_1, x_2 \longmapsto q_2] = [x_1 \longmapsto p_1 \cdot q_1, x_2 \longmapsto p_2 \cdot q_2]$.

(24) For every function F from I into the morphisms of C holds $F \cdot f = F \cdot (I \longmapsto f)$.

(25) For every function F from I into the morphisms of C holds $f \cdot F = (I \longmapsto f) \cdot F$.

3. Retractions and coretractions

Let us consider C and let l_1 be a morphism of C. We say that l_1 is retraction if and only if:

(Def. 10) There exists g such that $\mathrm{cod}\, g = \mathrm{dom}\, l_1$ and $l_1 \cdot g = \mathrm{id}_{\mathrm{cod}\, l_1}$.

We say that l_1 is coretraction if and only if:

(Def. 11) There exists g such that $\mathrm{dom}\, g = \mathrm{cod}\, l_1$ and $g \cdot l_1 = \mathrm{id}_{\mathrm{dom}\, l_1}$.

We now state a number of propositions:

(26) If f is retraction, then f is epi.

(27) If f is coretraction, then f is monic.

(28) If f is retraction and g is retraction and $\mathrm{dom}\, g = \mathrm{cod}\, f$, then $g \cdot f$ is retraction.

(29) If f is coretraction and g is coretraction and $\mathrm{dom}\, g = \mathrm{cod}\, f$, then $g \cdot f$ is coretraction.

(30) If $\mathrm{dom}\, g = \mathrm{cod}\, f$ and $g \cdot f$ is retraction, then g is retraction.

(31) If $\mathrm{dom}\, g = \mathrm{cod}\, f$ and $g \cdot f$ is coretraction, then f is coretraction.

(32) If f is retraction and monic, then f is invertible.

(33) If f is coretraction and epi, then f is invertible.

(34) f is invertible iff f is retraction and coretraction.

(35) For every functor T from C to D such that f is retraction holds $T(f)$ is retraction.

(36) For every functor T from C to D such that f is coretraction holds $T(f)$ is coretraction.

(37) f is retraction iff f^{op} is coretraction.

(38) f is coretraction iff f^{op} is retraction.

4. Morphisms determined by a terminal object

Let us consider C, a, b. Let us assume that b is terminal.

(Def. 12) $|^{a}b$ is a morphism from a to b.

We now state three propositions:

(39) If b is terminal, then $\mathrm{dom}\,|^{a}b = a$ and $\mathrm{cod}\,|^{a}b = b$.

(40) If b is terminal and $\mathrm{dom}\,f = a$ and $\mathrm{cod}\,f = b$, then $|^{a}b = f$.

(41) For every morphism f from a to b such that b is terminal holds $|^{a}b = f$.

5. Morphisms determined by an initial object

Let us consider C, a, b. Let us assume that a is initial.

(Def. 13) $\mathrm{init}(a,b)$ is a morphism from a to b.

One can prove the following three propositions:

(42) If a is initial, then $\mathrm{dom}\,\mathrm{init}(a,b) = a$ and $\mathrm{cod}\,\mathrm{init}(a,b) = b$.

(43) If a is initial and $\mathrm{dom}\,f = a$ and $\mathrm{cod}\,f = b$, then $\mathrm{init}(a,b) = f$.

(44) For every morphism f from a to b such that a is initial holds $\mathrm{init}(a,b) = f$.

6. Products

Let us consider C, a, I. A function from I into the morphisms of C is said to be a projections family from a onto I if:

(Def. 14) $\mathrm{dom}_{\kappa}\,\mathrm{it}(\kappa) = I \longmapsto a$.

One can prove the following propositions:

(45) For every projections family F from a onto I such that $x \in I$ holds $\mathrm{dom}(F_x) = a$.

(46) Every function from $\emptyset$ into the morphisms of C is a projections family from a onto $\emptyset$.

(47) If $\mathrm{dom}\,f = a$, then $\{y\} \longmapsto f$ is a projections family from a onto $\{y\}$.

(48) If $\mathrm{dom}\,p_1 = a$ and $\mathrm{dom}\,p_2 = a$, then $[x_1 \longmapsto p_1, x_2 \longmapsto p_2]$ is a projections family from a onto $\{x_1, x_2\}$.

(50)[3] Let F be a projections family from a onto I. If $\mathrm{cod}\,f = a$, then $F \cdot f$ is a projections family from $\mathrm{dom}\,f$ onto I.

(51) Let F be a function from I into the morphisms of C and G be a projections family from a onto I. If $\mathrm{dom}_{\kappa}\,F(\kappa) = \mathrm{cod}_{\kappa}\,G(\kappa)$, then $F \cdot G$ is a projections family from a onto I.

(52) For every projections family F from $\mathrm{cod}\,f$ onto I holds $f^{\mathrm{op}} \cdot F^{\mathrm{op}} = (F \cdot f)^{\mathrm{op}}$.

Let us consider C, a, I and let F be a function from I into the morphisms of C. We say that a is a product w.r.t. F if and only if the conditions (Def. 15) are satisfied.

(Def. 15)(i) F is a projections family from a onto I, and

(ii) for every b and for every projections family F' from b onto I such that $\mathrm{cod}_{\kappa}\,F(\kappa) = \mathrm{cod}_{\kappa}\,F'(\kappa)$ there exists h such that $h \in \mathrm{hom}(b,a)$ and for every k such that $k \in \mathrm{hom}(b,a)$ holds for every x such that $x \in I$ holds $F_x \cdot k = F'_x$ iff $h = k$.

[3] The proposition (49) has been removed.

One can prove the following propositions:

(53) Let F be a projections family from c onto I and F' be a projections family from d onto I. Suppose c is a product w.r.t. F and d is a product w.r.t. F' and $\mathrm{cod}_\kappa F(\kappa) = \mathrm{cod}_\kappa F'(\kappa)$. Then c and d are isomorphic.

(54) Let F be a projections family from c onto I. Suppose c is a product w.r.t. F and for all x_1, x_2 such that $x_1 \in I$ and $x_2 \in I$ holds $\hom(\mathrm{cod}(F_{x_1}), \mathrm{cod}(F_{x_2})) \neq \emptyset$. Let given x. If $x \in I$, then F_x is retraction.

(55) For every function F from $\emptyset$ into the morphisms of C holds a is a product w.r.t. F iff a is terminal.

(56) Let F be a projections family from a onto I. Suppose a is a product w.r.t. F and $\mathrm{dom}\, f = b$ and $\mathrm{cod}\, f = a$ and f is invertible. Then b is a product w.r.t. $F \cdot f$.

(57) a is a product w.r.t. $\{y\} \longmapsto \mathrm{id}_a$.

(58) Let F be a projections family from a onto I. Suppose a is a product w.r.t. F and for every x such that $x \in I$ holds $\mathrm{cod}(F_x)$ is terminal. Then a is terminal.

Let us consider C, c, p_1, p_2. We say that c is a product w.r.t. p_1 and p_2 if and only if the conditions (Def. 16) are satisfied.

(Def. 16)(i) $\mathrm{dom}\, p_1 = c$,

(ii) $\mathrm{dom}\, p_2 = c$, and

(iii) for all d, f, g such that $f \in \hom(d, \mathrm{cod}\, p_1)$ and $g \in \hom(d, \mathrm{cod}\, p_2)$ there exists h such that $h \in \hom(d, c)$ and for every k such that $k \in \hom(d, c)$ holds $p_1 \cdot k = f$ and $p_2 \cdot k = g$ iff $h = k$.

Next we state several propositions:

(59) If $x_1 \neq x_2$, then c is a product w.r.t. p_1 and p_2 iff c is a product w.r.t. $[x_1 \longmapsto p_1, x_2 \longmapsto p_2]$.

(60) Suppose $\hom(c, a) \neq \emptyset$ and $\hom(c, b) \neq \emptyset$. Let p_1 be a morphism from c to a and p_2 be a morphism from c to b. Then c is a product w.r.t. p_1 and p_2 if and only if for every d such that $\hom(d, a) \neq \emptyset$ and $\hom(d, b) \neq \emptyset$ holds $\hom(d, c) \neq \emptyset$ and for every morphism f from d to a and for every morphism g from d to b there exists a morphism h from d to c such that for every morphism k from d to c holds $p_1 \cdot k = f$ and $p_2 \cdot k = g$ iff $h = k$.

(61) Suppose c is a product w.r.t. p_1 and p_2 and d is a product w.r.t. q_1 and q_2 and $\mathrm{cod}\, p_1 = \mathrm{cod}\, q_1$ and $\mathrm{cod}\, p_2 = \mathrm{cod}\, q_2$. Then c and d are isomorphic.

(62) If c is a product w.r.t. p_1 and p_2 and $\hom(\mathrm{cod}\, p_1, \mathrm{cod}\, p_2) \neq \emptyset$ and $\hom(\mathrm{cod}\, p_2, \mathrm{cod}\, p_1) \neq \emptyset$, then p_1 is retraction and p_2 is retraction.

(63) If c is a product w.r.t. p_1 and p_2 and $h \in \hom(c, c)$ and $p_1 \cdot h = p_1$ and $p_2 \cdot h = p_2$, then $h = \mathrm{id}_c$.

(64) If c is a product w.r.t. p_1 and p_2 and $\mathrm{dom}\, f = d$ and $\mathrm{cod}\, f = c$ and f is invertible, then d is a product w.r.t. $p_1 \cdot f$ and $p_2 \cdot f$.

(65) If c is a product w.r.t. p_1 and p_2 and $\mathrm{cod}\, p_2$ is terminal, then c and $\mathrm{cod}\, p_1$ are isomorphic.

(66) If c is a product w.r.t. p_1 and p_2 and $\mathrm{cod}\, p_1$ is terminal, then c and $\mathrm{cod}\, p_2$ are isomorphic.

7. Coproducts

Let us consider C, c, I. A function from I into the morphisms of C is said to be an injections family into c on I if:

(Def. 17) $\mathrm{cod}_\kappa \mathrm{it}(\kappa) = I \longmapsto c$.

One can prove the following propositions:

(67) For every injections family F into c on I such that $x \in I$ holds $\mathrm{cod}(F_x) = c$.

(68) Every function from $\emptyset$ into the morphisms of C is an injections family into a on $\emptyset$.

(69) If $\mathrm{cod}\, f = a$, then $\{y\} \longmapsto f$ is an injections family into a on $\{y\}$.

(70) If $\mathrm{cod}\, p_1 = c$ and $\mathrm{cod}\, p_2 = c$, then $[x_1 \longmapsto p_1, x_2 \longmapsto p_2]$ is an injections family into c on $\{x_1, x_2\}$.

(72)[4] For every injections family F into b on I such that $\mathrm{dom}\, f = b$ holds $f \cdot F$ is an injections family into $\mathrm{cod}\, f$ on I.

(73) Let F be an injections family into b on I and G be a function from I into the morphisms of C. If $\mathrm{dom}_\kappa F(\kappa) = \mathrm{cod}_\kappa G(\kappa)$, then $F \cdot G$ is an injections family into b on I.

(74) Let F be a function from I into the morphisms of C. Then F is a projections family from c onto I if and only if F^{op} is an injections family into c^{op} on I.

(75) Let F be a function from I into the morphisms of C^{op} and c be an object of C^{op}. Then F is an injections family into c on I if and only if ${}^{\mathrm{op}}F$ is a projections family from ${}^{\mathrm{op}}c$ onto I.

(76) For every injections family F into $\mathrm{dom}\, f$ on I holds $F^{\mathrm{op}} \cdot f^{\mathrm{op}} = (f \cdot F)^{\mathrm{op}}$.

Let us consider C, c, I and let F be a function from I into the morphisms of C. We say that c is a coproduct w.r.t. F if and only if the conditions (Def. 18) are satisfied.

(Def. 18)(i) F is an injections family into c on I, and

(ii) for every d and for every injections family F' into d on I such that $\mathrm{dom}_\kappa F(\kappa) = \mathrm{dom}_\kappa F'(\kappa)$ there exists h such that $h \in \mathrm{hom}(c,d)$ and for every k such that $k \in \mathrm{hom}(c,d)$ holds for every x such that $x \in I$ holds $k \cdot F_x = F'_x$ iff $h = k$.

Next we state several propositions:

(77) Let F be a function from I into the morphisms of C. Then c is a product w.r.t. F if and only if c^{op} is a coproduct w.r.t. F^{op}.

(78) Let F be an injections family into c on I and F' be an injections family into d on I. Suppose c is a coproduct w.r.t. F and d is a coproduct w.r.t. F' and $\mathrm{dom}_\kappa F(\kappa) = \mathrm{dom}_\kappa F'(\kappa)$. Then c and d are isomorphic.

(79) Let F be an injections family into c on I. Suppose c is a coproduct w.r.t. F and for all x_1, x_2 such that $x_1 \in I$ and $x_2 \in I$ holds $\mathrm{hom}(\mathrm{dom}(F_{x_1}), \mathrm{dom}(F_{x_2})) \neq \emptyset$. Let given x. If $x \in I$, then F_x is coretraction.

(80) Let F be an injections family into a on I. Suppose a is a coproduct w.r.t. F and $\mathrm{dom}\, f = a$ and $\mathrm{cod}\, f = b$ and f is invertible. Then b is a coproduct w.r.t. $f \cdot F$.

(81) For every injections family F into a on $\emptyset$ holds a is a coproduct w.r.t. F iff a is initial.

(82) a is a coproduct w.r.t. $\{y\} \longmapsto \mathrm{id}_a$.

(83) Let F be an injections family into a on I. Suppose a is a coproduct w.r.t. F and for every x such that $x \in I$ holds $\mathrm{dom}(F_x)$ is initial. Then a is initial.

[4] The proposition (71) has been removed.

Let us consider C, c, i_1, i_2. We say that c is a coproduct w.r.t. i_1 and i_2 if and only if the conditions (Def. 19) are satisfied.

(Def. 19)(i) $\operatorname{cod} i_1 = c$,

(ii) $\operatorname{cod} i_2 = c$, and

(iii) for all d, f, g such that $f \in \hom(\operatorname{dom} i_1, d)$ and $g \in \hom(\operatorname{dom} i_2, d)$ there exists h such that $h \in \hom(c, d)$ and for every k such that $k \in \hom(c, d)$ holds $k \cdot i_1 = f$ and $k \cdot i_2 = g$ iff $h = k$.

We now state several propositions:

(84) c is a product w.r.t. p_1 and p_2 iff $c^{\rm op}$ is a coproduct w.r.t. ${p_1}^{\rm op}$ and ${p_2}^{\rm op}$.

(85) If $x_1 \neq x_2$, then c is a coproduct w.r.t. i_1 and i_2 iff c is a coproduct w.r.t. $[x_1 \longmapsto i_1, x_2 \longmapsto i_2]$.

(86) Suppose c is a coproduct w.r.t. i_1 and i_2 and d is a coproduct w.r.t. j_1 and j_2 and $\operatorname{dom} i_1 = \operatorname{dom} j_1$ and $\operatorname{dom} i_2 = \operatorname{dom} j_2$. Then c and d are isomorphic.

(87) Suppose $\hom(a, c) \neq \emptyset$ and $\hom(b, c) \neq \emptyset$. Let i_1 be a morphism from a to c and i_2 be a morphism from b to c. Then c is a coproduct w.r.t. i_1 and i_2 if and only if for every d such that $\hom(a, d) \neq \emptyset$ and $\hom(b, d) \neq \emptyset$ holds $\hom(c, d) \neq \emptyset$ and for every morphism f from a to d and for every morphism g from b to d there exists a morphism h from c to d such that for every morphism k from c to d holds $k \cdot i_1 = f$ and $k \cdot i_2 = g$ iff $h = k$.

(88) If c is a coproduct w.r.t. i_1 and i_2 and $\hom(\operatorname{dom} i_1, \operatorname{dom} i_2) \neq \emptyset$ and $\hom(\operatorname{dom} i_2, \operatorname{dom} i_1) \neq \emptyset$, then i_1 is coretraction and i_2 is coretraction.

(89) If c is a coproduct w.r.t. i_1 and i_2 and $h \in \hom(c, c)$ and $h \cdot i_1 = i_1$ and $h \cdot i_2 = i_2$, then $h = \mathrm{id}_c$.

(90) Suppose c is a coproduct w.r.t. i_1 and i_2 and $\operatorname{dom} f = c$ and $\operatorname{cod} f = d$ and f is invertible. Then d is a coproduct w.r.t. $f \cdot i_1$ and $f \cdot i_2$.

(91) If c is a coproduct w.r.t. i_1 and i_2 and $\operatorname{dom} i_2$ is initial, then $\operatorname{dom} i_1$ and c are isomorphic.

(92) If c is a coproduct w.r.t. i_1 and i_2 and $\operatorname{dom} i_1$ is initial, then $\operatorname{dom} i_2$ and c are isomorphic.

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