JOURNAL OF FORMALIZED MATHEMATICS
Volume 15, Released 2003, Published 2003
Inst. of Computer Science, Univ. of Białystok

Propositional Calculus for Boolean Valued Functions. Part VII

Shunichi Kobayashi
Matsumoto University
Nagano

Summary. In this paper, we proved some elementary propositional calculus formulae for Boolean valued functions.

MML Identifier: BVFUNC25.

WWW: http://mizar.org/JFM/Vol15/bvfunc25.html

The articles [4], [3], [2], and [1] provide the notation and terminology for this paper.

We use the following convention: Y is a non empty set and a, b, c, d are elements of $Boolean^Y$.

We now state a number of propositions:

(1) $\neg(a \Rightarrow b) = a \wedge \neg b$.

(2) $\neg b \Rightarrow \neg a \Rightarrow a \Rightarrow b = true(Y)$.

(3) $a \Rightarrow b = \neg b \Rightarrow \neg a$.

(4) $a \Leftrightarrow b = \neg a \Leftrightarrow \neg b$.

(5) $a \Rightarrow b = a \Rightarrow a \wedge b$.

(6) $a \Leftrightarrow b = a \vee b \Rightarrow a \wedge b$.

(7) $a \Leftrightarrow \neg a = false(Y)$.

(8) $a \Rightarrow b \Rightarrow c = b \Rightarrow a \Rightarrow c$.

(9) $a \Rightarrow b \Rightarrow c = a \Rightarrow b \Rightarrow a \Rightarrow c$.

(10) $a \Leftrightarrow b = a \oplus \neg b$.

(11) $a \wedge (b \oplus c) = a \wedge b \oplus a \wedge c$.

(12) $a \Leftrightarrow b = \neg(a \oplus b)$.

(13) $a \oplus a = false(Y)$.

(14) $a \oplus \neg a = true(Y)$.

(15) $a \Rightarrow b \Rightarrow b \Rightarrow a = b \Rightarrow a$.

(16) $(a \vee b) \wedge (\neg a \vee \neg b) = \neg a \wedge b \vee a \wedge \neg b$.

(17) $a \wedge b \vee \neg a \wedge \neg b = (\neg a \vee b) \wedge (a \vee \neg b)$.

© Association of Mizar Users

(18) $a \oplus (b \oplus c) = (a \oplus b) \oplus c$.

(19) $a \Leftrightarrow b \Leftrightarrow c = a \Leftrightarrow b \Leftrightarrow c$.

(20) $\neg\neg a \Rightarrow a = true(Y)$.

(21) $(a \Rightarrow b) \wedge a \Rightarrow b = true(Y)$.

(22) $a \Rightarrow \neg a \Rightarrow a = true(Y)$.

(23) $\neg a \Rightarrow a \Leftrightarrow a = true(Y)$.

(24) $a \vee (a \Rightarrow b) = true(Y)$.

(25) $(a \Rightarrow b) \vee (c \Rightarrow a) = true(Y)$.

(26) $(a \Rightarrow b) \vee (\neg a \Rightarrow b) = true(Y)$.

(27) $(a \Rightarrow b) \vee (a \Rightarrow \neg b) = true(Y)$.

(28) $\neg a \Rightarrow \neg b \Leftrightarrow b \Rightarrow a = true(Y)$.

(29) $a \Rightarrow b \Rightarrow a \Rightarrow c \Rightarrow b \Rightarrow b = true(Y)$.

(30) $a \Rightarrow b = a \Leftrightarrow a \wedge b$.

(31) $a \Rightarrow b = true(Y)$ and $b \Rightarrow a = true(Y)$ iff $a = b$.

(32) $a = \neg a \Rightarrow a$.

(33) $a \Rightarrow a \Rightarrow b \Rightarrow a = true(Y)$.

(34) $a = a \Rightarrow b \Rightarrow a$.

(35) $a = (b \Rightarrow a) \wedge (\neg b \Rightarrow a)$.

(36) $a \wedge b = \neg(a \Rightarrow \neg b)$.

(37) $a \vee b = \neg a \Rightarrow b$.

(38) $a \vee b = a \Rightarrow b \Rightarrow b$.

(39) $a \Rightarrow b \Rightarrow a \Rightarrow a = true(Y)$.

(40) $a \Rightarrow b \Rightarrow c \Rightarrow d \Rightarrow b \Rightarrow a \Rightarrow d \Rightarrow c = true(Y)$.

(41) $(a \Rightarrow b) \wedge a \wedge c \Rightarrow b = true(Y)$.

(42) $b \Rightarrow c \Rightarrow a \wedge b \Rightarrow c = true(Y)$.

(43) $a \wedge b \Rightarrow c \Rightarrow a \wedge b \Rightarrow c \wedge b = true(Y)$.

(44) $a \Rightarrow b \Rightarrow a \wedge c \Rightarrow b \wedge c = true(Y)$.

(45) $(a \Rightarrow b) \wedge (a \wedge c) \Rightarrow b \wedge c = true(Y)$.

(46) $a \wedge (a \Rightarrow b) \wedge (b \Rightarrow c) \Subset c$.

(47) $(a \vee b) \wedge (a \Rightarrow c) \wedge (b \Rightarrow c) \Subset \neg a \Rightarrow b \vee c$.

ACKNOWLEDGMENTS

This research was partially supported by the research funds of the University of Matsumoto.

REFERENCES

[1] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/bvfunc_1.html.

[2] Andrzej Trybulec. Function domains and Frænkel operator. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/fraenkel.html.

[3] Edmund Woronowicz. Interpretation and satisfiability in the first order logic. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/valuat_1.html.

[4] Edmund Woronowicz. Many-argument relations. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/margrel1.html.

Received February 6, 2003

Published January 2, 2004