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Six Variable Predicate Calculus for Boolean Valued Functions. Part I

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The articles [11], [10], [2], [12], [8], [13], [1], [3], [4], [9], [6], [5], and [7] provide the notation and terminology for this paper.

We use the following convention: Y is a non empty set, G is a subset of PARTITIONS(Y), and A, B, C, D, E, F are partitions of Y.

We now state a number of propositions:

(1) Suppose that $G = \{A,B,C,D,E,F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(A,G) = B \wedge C \wedge D \wedge E \wedge F$.

(2) Suppose that G is independent and $G = \{A,B,C,D,E,F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(B,G) = A \wedge C \wedge D \wedge E \wedge F$.

(3) Suppose that G is independent and $G = \{A,B,C,D,E,F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(C,G) = A \wedge B \wedge D \wedge E \wedge F$.

(4) Suppose that G is independent and $G = \{A,B,C,D,E,F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(D,G) = A \wedge B \wedge C \wedge E \wedge F$.

(5) Suppose that G is independent and $G = \{A,B,C,D,E,F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(E,G) = A \wedge B \wedge C \wedge D \wedge F$.

(6) Suppose that G is independent and $G = \{A,B,C,D,E,F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(F,G) = A \wedge B \wedge C \wedge D \wedge E$.

(7) Let A, B, C, D, E, F be sets, h be a function, and A', B', C', D', E', F' be sets. Suppose that $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$ and

$h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (A \dot{\longmapsto} A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$ and $h(F) = F'$.

(8) Let A, B, C, D, E, F be sets, h be a function, and A', B', C', D', E', F' be sets. If $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (A \dot{\longmapsto} A')$, then $\mathrm{dom}\, h = \{A, B, C, D, E, F\}$.

(9) Let A, B, C, D, E, F be sets, h be a function, and A', B', C', D', E', F' be sets. Suppose that $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$ and $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (A \dot{\longmapsto} A')$. Then $\mathrm{rng}\, h = \{h(A), h(B), h(C), h(D), h(E), h(F)\}$.

(10) Let G be a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F be partitions of Y, z, u be elements of Y, and h be a function. Suppose that G is independent and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E \wedge F)$ meets $\mathrm{EqClass}(z, A)$.

(11) Let G be a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F be partitions of Y, z, u be elements of Y, and h be a function. Suppose that G is independent and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$ and $\mathrm{EqClass}(z, C \wedge D \wedge E \wedge F) = \mathrm{EqClass}(u, C \wedge D \wedge E \wedge F)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G))$ meets $\mathrm{EqClass}(z, \mathrm{CompF}(B, G))$.

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