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Five Variable Predicate Calculus for Boolean Valued Functions. Part I

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The articles [11], [10], [2], [12], [8], [13], [1], [3], [4], [9], [6], [5], and [7] provide the notation and terminology for this paper.

We follow the rules: Y is a non empty set, G is a subset of PARTITIONS(Y), and A, B, C, D, E are partitions of Y.

We now state a number of propositions:

(1) If $G = \{A,B,C,D,E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$, then CompF$(A,G) = B \wedge C \wedge D \wedge E$.

(2) Suppose G is independent and $G = \{A,B,C,D,E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then CompF$(B,G) = A \wedge C \wedge D \wedge E$.

(3) Suppose G is independent and $G = \{A,B,C,D,E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then CompF$(C,G) = A \wedge B \wedge D \wedge E$.

(4) Suppose G is independent and $G = \{A,B,C,D,E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then CompF$(D,G) = A \wedge B \wedge C \wedge E$.

(5) Suppose G is independent and $G = \{A,B,C,D,E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then CompF$(E,G) = A \wedge B \wedge C \wedge D$.

(6) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. Suppose $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$ and $h = (B \mapsto B') + \cdot (C \mapsto C') + \cdot (D \mapsto D') + \cdot (E \mapsto E') + \cdot (A \mapsto A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$.

(7) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. If $h = (B \mapsto B') + \cdot (C \mapsto C') + \cdot (D \mapsto D') + \cdot (E \mapsto E') + \cdot (A \mapsto A')$, then $\operatorname{dom} h = \{A,B,C,D,E\}$.

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(8) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. If $h = (B \mapsto B') + \cdot (C \mapsto C') + \cdot (D \mapsto D') + \cdot (E \mapsto E') + \cdot (A \mapsto A')$, then $\operatorname{rng} h = \{h(A), h(B), h(C), h(D), h(E)\}$.

(9) Let G be a subset of PARTITIONS(Y), A, B, C, D, E be partitions of Y, z, u be elements of Y, and h be a function. Suppose G is independent and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then EqClass$(u, B \wedge C \wedge D \wedge E)$ meets EqClass(z, A).

(10) Let G be a subset of PARTITIONS(Y), A, B, C, D, E be partitions of Y, and z, u be elements of Y. Suppose that G is independent and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$ and EqClass$(z, C \wedge D \wedge E) =$ EqClass$(u, C \wedge D \wedge E)$. Then EqClass$(u, \text{CompF}(A, G))$ meets EqClass$(z, \text{CompF}(B, G))$.

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