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Four Variable Predicate Calculus for Boolean Valued Functions. Part I

Shunichi Kobayashi
Ueda Multimedia Information Center
Nagano

Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The articles [11], [10], [2], [13], [8], [14], [1], [12], [3], [4], [16], [15], [9], [6], [5], and [7] provide the notation and terminology for this paper.

1. Preliminaries

For simplicity, we adopt the following convention: Y denotes a non empty set, a denotes an element of $Boolean^Y$, G denotes a subset of PARTITIONS(Y), and A, B, C, D denote partitions of Y.

One can prove the following propositions:

(1) Let h be a function and A', B', C', D' be sets. Suppose $G = \{A,B,C,D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$ and $h = (B \longmapsto B') + \cdot (C \longmapsto C') + \cdot (D \longmapsto D') + \cdot (A \longmapsto A')$. Then $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$.

(2) Let A, B, C, D be sets, h be a function, and A', B', C', D' be sets. If $h = (B \longmapsto B') + \cdot (C \longmapsto C') + \cdot (D \longmapsto D') + \cdot (A \longmapsto A')$, then $\operatorname{dom} h = \{A,B,C,D\}$.

(3) For every function h and for all sets A', B', C', D' such that $G = \{A,B,C,D\}$ and $h = (B \longmapsto B') + \cdot (C \longmapsto C') + \cdot (D \longmapsto D') + \cdot (A \longmapsto A')$ holds $\operatorname{rng} h = \{h(A), h(B), h(C), h(D)\}$.

(4) Let z, u be elements of Y and h be a function. Suppose G is independent and $G = \{A,B,C,D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$. Then EqClass$(u, B \wedge C \wedge D)$ meets EqClass(z,A).

(5) Let z, u be elements of Y. Suppose G is independent and $G = \{A,B,C,D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$ and EqClass$(z, C \wedge D) =$ EqClass$(u, C \wedge D)$. Then EqClass$(u,$ CompF$(A,G))$ meets EqClass$(z,$ CompF$(B,G))$.

2. Four Variable Predicate Calculus

We now state two propositions:

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(20)[1] If G is independent and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg\forall_{a,A}G,B} G \Subset \neg\forall_{\forall_{a,B}G,A} G$.

(23)[2] If G is independent and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\exists_{\neg a,A}G,B} G \Subset \neg\forall_{\forall_{a,B}G,A} G$.

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[1] The propositions (6)–(19) have been removed.

[2] The propositions (21) and (22) have been removed.