JOURNAL OF FORMALIZED MATHEMATICS
Volume 11, Released 1999, Published 2003
Inst. of Computer Science, Univ. of Białystok

Predicate Calculus for Boolean Valued Functions. Part V

Shunichi Kobayashi
Shinshu University
Nagano

Yatsuka Nakamura
Shinshu University
Nagano

Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

MML Identifier: `BVFUNC13`.

WWW: `http://mizar.org/JFM/Vol11/bvfunc13.html`

The articles [5], [7], [6], [4], [3], [1], and [2] provide the notation and terminology for this paper.

For simplicity, we follow the rules: Y denotes a non empty set, a denotes an element of $Boolean^Y$, G denotes a subset of PARTITIONS(Y), and A, B denote partitions of Y.

Next we state a number of propositions:

(1) If G is independent, then $\forall_{\neg\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(2) $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(3) $\forall_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(4) If G is independent, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(6)[1] If G is independent, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(7) If G is independent, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(9)[2] If G is independent, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(10) If G is independent, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(11) $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

(14)[3] If G is independent, then $\neg\exists_{\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(15) If G is independent, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(16) $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(17) If G is independent, then $\neg\exists_{\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

[1] The proposition (5) has been removed.
[2] The proposition (8) has been removed.
[3] The propositions (12) and (13) have been removed.

(18) If G is independent, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(19) $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(20) If G is independent, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(21) If G is independent, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(22) $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\neg\exists_{a,B}G,A}G$.

(23) If G is independent, then $\neg\exists_{\exists_{a,A}G,B}G = \forall_{\neg\exists_{a,B}G,A}G$.

(24) If G is independent, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(25) $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(26) If G is independent, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(27) If G is independent, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(28) $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\forall_{\neg a,B}G,A}G$.

(29) If G is independent, then $\neg\exists_{\exists_{a,A}G,B}G = \forall_{\forall_{\neg a,B}G,A}G$.

(30) If G is independent, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(31) $\forall_{\neg\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(32) $\forall_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

(33) If G is independent, then $\forall_{\neg\exists_{a,A}G,B}G = \neg\exists_{\exists_{a,B}G,A}G$.

(34) If G is independent, then $\exists_{\neg\forall_{a,A}G,B}G = \exists_{\neg\forall_{a,B}G,A}G$.

(35) If G is independent, then $\forall_{\neg\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(36) If G is independent, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(37) $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(38) If G is independent, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(39) If G is independent, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(40) $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\neg\exists_{a,B}G,A}G$.

(41) If G is independent, then $\forall_{\neg\exists_{a,A}G,B}G = \forall_{\neg\exists_{a,B}G,A}G$.

(42) If G is independent, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(43) $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(44) If G is independent, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(45) If G is independent, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(46) $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\forall_{\neg a,B}G,A}G$.

(47) If G is independent, then $\forall_{\neg\exists_{a,A}G,B}G = \forall_{\forall_{\neg a,B}G,A}G$.

(48) If G is independent, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(49) If G is independent, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(50) $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

(51) If G is independent, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\exists_{a,B}G,A}G$.

(52) If G is independent, then $\exists_{\exists_{\neg a,A}G,B}G \Subset \exists_{\neg \forall_{a,B}G,A}G$.

(53) If G is independent, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \exists_{\neg \forall_{a,B}G,A}G$.

(54) If G is independent, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg \forall_{a,B}G,A}G$.

(55) $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg \forall_{a,B}G,A}G$.

(56) If G is independent, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg \forall_{a,B}G,A}G$.

(57) If G is independent, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg \forall_{a,B}G,A}G$.

(58) $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg \exists_{a,B}G,A}G$.

(59) If G is independent, then $\forall_{\forall_{\neg a,A}G,B}G = \forall_{\neg \exists_{a,B}G,A}G$.

(61)[4] $\forall_{\exists_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(62) If G is independent, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

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Received August 17, 1999

Published January 2, 2004

[4] The proposition (60) has been removed.