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Propositional Calculus for Boolean Valued Functions. Part VI

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Summary. In this paper, we proved some elementary propositional calculus formulae for Boolean valued functions.

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The articles [3], [5], [4], [2], and [1] provide the notation and terminology for this paper.

In this paper Y denotes a non empty set.

One can prove the following propositions:

(1) For all elements a, b, c of $Boolean^Y$ holds $a \wedge b \vee b \wedge c \vee c \wedge a = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.

(2) For all elements a, b, c of $Boolean^Y$ holds $a \wedge \neg b \vee b \wedge \neg c \vee c \wedge \neg a = b \wedge \neg a \vee c \wedge \neg b \vee a \wedge \neg c$.

(3) For all elements a, b, c of $Boolean^Y$ holds $(a \vee \neg b) \wedge (b \vee \neg c) \wedge (c \vee \neg a) = (b \vee \neg a) \wedge (c \vee \neg b) \wedge (a \vee \neg c)$.

(4) For all elements a, b, c of $Boolean^Y$ such that $c \Rightarrow a = true(Y)$ and $c \Rightarrow b = true(Y)$ holds $c \Rightarrow a \vee b = true(Y)$.

(5) For all elements a, b, c of $Boolean^Y$ such that $a \Rightarrow c = true(Y)$ and $b \Rightarrow c = true(Y)$ holds $a \wedge b \Rightarrow c = true(Y)$.

(6) For all elements a_1, a_2, b_1, b_2, c_1, c_2 of $Boolean^Y$ holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge (a_1 \vee b_1 \vee c_1) \in a_2 \vee b_2 \vee c_2$.

(7) For all elements a_1, a_2, b_1, b_2 of $Boolean^Y$ holds $(a_1 \Rightarrow b_1) \wedge (a_2 \Rightarrow b_2) \wedge (a_1 \vee a_2) \wedge \neg(b_1 \wedge b_2) = (b_1 \Rightarrow a_1) \wedge (b_2 \Rightarrow a_2) \wedge (b_1 \vee b_2) \wedge \neg(a_1 \wedge a_2)$.

(8) For all elements a, b, c, d of $Boolean^Y$ holds $(a \vee b) \wedge (c \vee d) = a \wedge c \vee a \wedge d \vee b \wedge c \vee b \wedge d$.

(9) For all elements a_1, a_2, b_1, b_2, b_3 of $Boolean^Y$ holds $a_1 \wedge a_2 \vee b_1 \wedge b_2 \wedge b_3 = (a_1 \vee b_1) \wedge (a_1 \vee b_2) \wedge (a_1 \vee b_3) \wedge (a_2 \vee b_1) \wedge (a_2 \vee b_2) \wedge (a_2 \vee b_3)$.

(10) For all elements a, b, c, d of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow d) = (a \Rightarrow b \wedge c \wedge d) \wedge (b \Rightarrow c \wedge d) \wedge (c \Rightarrow d)$.

(11) For all elements a, b, c, d of $Boolean^Y$ holds $(a \Rightarrow c) \wedge (b \Rightarrow d) \wedge (a \vee b) \in c \vee d$.

(12) For all elements a, b, c of $Boolean^Y$ holds $(a \wedge b \Rightarrow \neg c) \wedge a \wedge c \in \neg b$.

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(13) For all elements a_1, a_2, a_3, b_1, b_2, b_3 of $Boolean^Y$ holds $a_1 \wedge a_2 \wedge a_3 \Rightarrow b_1 \vee b_2 \vee b_3 = \neg b_1 \wedge \neg b_2 \wedge a_3 \Rightarrow \neg a_1 \vee \neg a_2 \vee b_3$.

(14) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) = a \wedge b \wedge c \vee \neg a \wedge \neg b \wedge \neg c$.

(15) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) \wedge (a \vee b \vee c) = a \wedge b \wedge c$.

(16) For all elements a, b, c of $Boolean^Y$ holds $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) \wedge \neg(a \wedge b \wedge c) = \neg a \wedge b \wedge c \vee a \wedge \neg b \wedge c \vee a \wedge b \wedge \neg c$.

(17) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \wedge c$.

(18) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \vee b \Rightarrow c$.

(19) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \vee c$.

(20) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \vee \neg c$.

(21) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset b \Rightarrow c \vee a$.

(22) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset b \Rightarrow c \vee \neg a$.

(23) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b) \wedge (b \Rightarrow c \vee a)$.

(24) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c)$.

(25) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee c) \wedge (b \Rightarrow c \vee a)$.

(26) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c \vee a)$.

(27) For all elements a, b, c of $Boolean^Y$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c \vee \neg a)$.

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