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# On the Calculus of Binary Arithmetics

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**Summary.** In this paper, we have binary arithmetic and its related operations. We include some theorems concerning logical operators.

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The articles [3], [4], [2], and [1] provide the notation and terminology for this paper.

Let $x$, $y$ be boolean sets. The functor $x\ 'nand'\ y$ is defined by:

(Def. 1) $x\ 'nand'\ y = \neg(x \wedge y)$.

Let us notice that the functor $x\ 'nand'\ y$ is commutative.

Let $x$, $y$ be boolean sets. Note that $x\ 'nand'\ y$ is boolean.

Let $x$, $y$ be elements of *Boolean*. Then $x\ 'nand'\ y$ is an element of *Boolean*.

Let $x$, $y$ be boolean sets. The functor $x\ 'nor'\ y$ is defined by:

(Def. 2) $x\ 'nor'\ y = \neg(x \vee y)$.

Let us observe that the functor $x\ 'nor'\ y$ is commutative.

Let $x$, $y$ be boolean sets. One can check that $x\ 'nor'\ y$ is boolean.

Let $x$, $y$ be elements of *Boolean*. Then $x\ 'nor'\ y$ is an element of *Boolean*.

Let $x$, $y$ be boolean sets. The functor $x\ 'xnor'\ y$ is defined by:

(Def. 3) $x\ 'xnor'\ y = \neg(x \oplus y)$.

Let us observe that the functor $x\ 'xnor'\ y$ is commutative.

Let $x$, $y$ be boolean sets. One can verify that $x\ 'xnor'\ y$ is boolean.

Let $x$, $y$ be elements of *Boolean*. Then $x\ 'xnor'\ y$ is an element of *Boolean*.

In the sequel $x$, $y$, $z$, $w$ denote boolean sets.

The following propositions are true:

(1) $true\ 'nand'\ x = \neg x$.

(2) $false\ 'nand'\ x = true$.

(3) $x\ 'nand'\ x = \neg x$ and $\neg(x\ 'nand'\ x) = x$.

(4) $\neg(x\ 'nand'\ y) = x \wedge y$.

(5) $x\ 'nand'\ \neg x = true$ and $\neg(x\ 'nand'\ \neg x) = false$.

(6) $x\ 'nand'\ y \wedge z = \neg(x \wedge y \wedge z)$.

(7) $x\ 'nand'\ y \wedge z = x \wedge y\ 'nand'\ z$.



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(8) $x\ {}'\text{nand}'\ (y \vee z) = \neg(x \wedge y) \wedge \neg(x \wedge z)$.

(9) $x\ {}'\text{nand}'\ (y \oplus z) = x \wedge y \Leftrightarrow x \wedge z$.

(10) $\mathit{true}\ {}'\text{nor}'\ x = \mathit{false}$.

(11) $\mathit{false}\ {}'\text{nor}'\ x = \neg x$.

(12) $x\ {}'\text{nor}'\ x = \neg x$ and $\neg(x\ {}'\text{nor}'\ x) = x$.

(13) $\neg(x\ {}'\text{nor}'\ y) = x \vee y$.

(14) $x\ {}'\text{nor}'\ \neg x = \mathit{false}$ and $\neg(x\ {}'\text{nor}'\ \neg x) = \mathit{true}$.

(15) $x\ {}'\text{nor}'\ y \wedge z = \neg(x \vee y) \vee \neg(x \vee z)$.

(16) $x\ {}'\text{nor}'\ (y \vee z) = \neg(x \vee y \vee z)$.

(17) $\mathit{true}\ {}'\text{xnor}'\ x = x$.

(18) $\mathit{false}\ {}'\text{xnor}'\ x = \neg x$.

(19) $x\ {}'\text{xnor}'\ x = \mathit{true}$ and $\neg(x\ {}'\text{xnor}'\ x) = \mathit{false}$.

(20) $\neg(x\ {}'\text{xnor}'\ y) = x \oplus y$.

(21) $x\ {}'\text{xnor}'\ \neg x = \mathit{false}$ and $\neg(x\ {}'\text{xnor}'\ \neg x) = \mathit{true}$.

(22) $x \Subset y \Rightarrow z$ iff $x \wedge y \Subset z$.

(23) $x \Leftrightarrow y = (x \Rightarrow y) \wedge (y \Rightarrow x)$.

(24) $x \Leftrightarrow y = \mathit{true}$ iff $x \Rightarrow y = \mathit{true}$ and $y \Rightarrow x = \mathit{true}$.

(25) If $x \Rightarrow y = \mathit{true}$ and $y \Rightarrow x = \mathit{true}$, then $x = y$.

(26) If $x \Rightarrow y = \mathit{true}$ and $y \Rightarrow z = \mathit{true}$, then $x \Rightarrow z = \mathit{true}$.

(27) If $x \Leftrightarrow y = \mathit{true}$ and $y \Leftrightarrow z = \mathit{true}$, then $x \Leftrightarrow z = \mathit{true}$.

(28) $x \Rightarrow y = \neg y \Rightarrow \neg x$.

(29) $x \Leftrightarrow y = \neg x \Leftrightarrow \neg y$.

(30) If $x \Leftrightarrow y = \mathit{true}$ and $z \Leftrightarrow w = \mathit{true}$, then $x \wedge z \Leftrightarrow y \wedge w = \mathit{true}$.

(31) If $x \Leftrightarrow y = \mathit{true}$ and $z \Leftrightarrow w = \mathit{true}$, then $x \Rightarrow z \Leftrightarrow y \Rightarrow w = \mathit{true}$.

(32) If $x \Leftrightarrow y = \mathit{true}$ and $z \Leftrightarrow w = \mathit{true}$, then $x \vee z \Leftrightarrow y \vee w = \mathit{true}$.

(33) If $x \Leftrightarrow y = \mathit{true}$ and $z \Leftrightarrow w = \mathit{true}$, then $x \Leftrightarrow z \Leftrightarrow y \Leftrightarrow w = \mathit{true}$.

(34) If $x = \mathit{true}$ and $x \Rightarrow y = \mathit{true}$, then $y = \mathit{true}$.

(35) If $y = \mathit{true}$, then $x \Rightarrow y = \mathit{true}$.

(36) If $\neg x = \mathit{true}$, then $x \Rightarrow y = \mathit{true}$.

(37) $x \Rightarrow x = \mathit{true}$.

(38) If $x \Rightarrow y = \mathit{true}$ and $x \Rightarrow \neg y = \mathit{true}$, then $\neg x = \mathit{true}$.

(39) $\neg x \Rightarrow x \Rightarrow x = \mathit{true}$.

(40) $x \Rightarrow y \Rightarrow \neg(y \wedge z) \Rightarrow \neg(x \wedge z) = \mathit{true}$.

(41) $x \Rightarrow y \Rightarrow y \Rightarrow z \Rightarrow x \Rightarrow z = \mathit{true}$.

(42) If $x \Rightarrow y = \mathit{true}$, then $y \Rightarrow z \Rightarrow x \Rightarrow z = \mathit{true}$.

(43) $y \Rightarrow x \Rightarrow y = \mathit{true}$.

(44) $x \Rightarrow y \Rightarrow z \Rightarrow y \Rightarrow z = \mathit{true}$.

(45) $y \Rightarrow y \Rightarrow x \Rightarrow x = \mathit{true}$.

(46) $z \Rightarrow y \Rightarrow x \Rightarrow y \Rightarrow z \Rightarrow x = \mathit{true}$.

(47) $y \Rightarrow z \Rightarrow x \Rightarrow y \Rightarrow x \Rightarrow z = \mathit{true}$.

(48) $y \Rightarrow y \Rightarrow z \Rightarrow y \Rightarrow z = \mathit{true}$.

(49) $x \Rightarrow y \Rightarrow z \Rightarrow x \Rightarrow y \Rightarrow x \Rightarrow z = \mathit{true}$.

(50) If $x = \mathit{true}$, then $x \Rightarrow y \Rightarrow y = \mathit{true}$.

(51) If $z \Rightarrow y \Rightarrow x = \mathit{true}$, then $y \Rightarrow z \Rightarrow x = \mathit{true}$.

(52) If $z \Rightarrow y \Rightarrow x = \mathit{true}$ and $y = \mathit{true}$, then $z \Rightarrow x = \mathit{true}$.

(53) If $z \Rightarrow y \Rightarrow x = \mathit{true}$ and $y = \mathit{true}$ and $z = \mathit{true}$, then $x = \mathit{true}$.

(54) If $y \Rightarrow y \Rightarrow z = \mathit{true}$, then $y \Rightarrow z = \mathit{true}$.

(55) If $x \Rightarrow y \Rightarrow z = \mathit{true}$, then $x \Rightarrow y \Rightarrow x \Rightarrow z = \mathit{true}$.

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