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Introduction to Arithmetics[1]

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The articles [7], [5], [11], [12], [3], [4], [6], [1], [2], [8], [9], and [10] provide the notation and terminology for this paper.

1. Main Block

One can prove the following propositions:

(1) $\mathbb{R}_+ \subseteq \mathbb{R}$.

(2) For every element x of $\mathbb{R}_+$ such that $x \neq \emptyset$ holds $\langle \emptyset, x \rangle \in \mathbb{R}$.

(3) For every set y such that $\langle \emptyset, y \rangle \in \mathbb{R}$ holds $y \neq \emptyset$.

(4) For all elements x, y of $\mathbb{R}_+$ holds $x - y \in \mathbb{R}$.

(5) $\mathbb{R}_+$ misses $[: \{\emptyset\}, \mathbb{R}_+ :]$.

2. Real Numbers

We now state three propositions:

(6) For all elements x, y of $\mathbb{R}_+$ such that $x - y = \emptyset$ holds $x = y$.

(7) It is not true that there exist sets a, b such that $\mathbf{1} = \langle a, b \rangle$.

(8) For all elements x, y, z of $\mathbb{R}_+$ such that $x \neq \emptyset$ and $x * y = x * z$ holds $y = z$.

3. ??????? Moved from XREAL_0 ?????????

Let x, y be elements of $\mathbb{R}$. The functor $+(x,y)$ yielding an element of $\mathbb{R}$ is defined as follows:

(Def. 2)[1](i) There exist elements x', y' of $\mathbb{R}_+$ such that $x = x'$ and $y = y'$ and $+(x,y) = x' + y'$ if $x \in \mathbb{R}_+$ and $y \in \mathbb{R}_+$,

(ii) there exist elements x', y' of $\mathbb{R}_+$ such that $x = x'$ and $y = \langle 0, y' \rangle$ and $+(x,y) = x' - y'$ if $x \in \mathbb{R}_+$ and $y \in [: \{0\}, \mathbb{R}_+ :]$,

(iii) there exist elements x', y' of $\mathbb{R}_+$ such that $x = \langle 0, x' \rangle$ and $y = y'$ and $+(x,y) = y' - x'$ if $y \in \mathbb{R}_+$ and $x \in [: \{0\}, \mathbb{R}_+ :]$,

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[1] The definition (Def. 1) has been removed.

(iv) there exist elements x', y' of $\mathbb{R}_+$ such that $x = \langle 0, x' \rangle$ and $y = \langle 0, y' \rangle$ and $+(x, y) = \langle 0, x' + y' \rangle$, otherwise.

Let us notice that the functor $+(x, y)$ is commutative. The functor $\cdot(x, y)$ yields an element of $\mathbb{R}$ and is defined as follows:

(Def. 3)(i) There exist elements x', y' of $\mathbb{R}_+$ such that $x = x'$ and $y = y'$ and $\cdot(x, y) = x' * y'$ if $x \in \mathbb{R}_+$ and $y \in \mathbb{R}_+$,

(ii) there exist elements x', y' of $\mathbb{R}_+$ such that $x = x'$ and $y = \langle 0, y' \rangle$ and $\cdot(x, y) = \langle 0, x' * y' \rangle$ if $x \in \mathbb{R}_+$ and $y \in [: \{0\}, \mathbb{R}_+ :]$ and $x \neq 0$,

(iii) there exist elements x', y' of $\mathbb{R}_+$ such that $x = \langle 0, x' \rangle$ and $y = y'$ and $\cdot(x, y) = \langle 0, y' * x' \rangle$ if $y \in \mathbb{R}_+$ and $x \in [: \{0\}, \mathbb{R}_+ :]$ and $y \neq 0$,

(iv) there exist elements x', y' of $\mathbb{R}_+$ such that $x = \langle 0, x' \rangle$ and $y = \langle 0, y' \rangle$ and $\cdot(x, y) = y' * x'$ if $x \in [: \{0\}, \mathbb{R}_+ :]$ and $y \in [: \{0\}, \mathbb{R}_+ :]$,

(v) $\cdot(x, y) = 0$, otherwise.

Let us note that the functor $\cdot(x, y)$ is commutative.

In the sequel x, y denote elements of $\mathbb{R}$.

Let x be an element of $\mathbb{R}$. The functor ${}^{\mathrm{op}}x$ yields an element of $\mathbb{R}$ and is defined as follows:

(Def. 4) $+(x, {}^{\mathrm{op}}x) = 0$.

Let us note that the functor ${}^{\mathrm{op}}x$ is involutive. The functor $\mathrm{inv}\,x$ yielding an element of $\mathbb{R}$ is defined by:

(Def. 5)(i) $\cdot(x, \mathrm{inv}\,x) = \mathbf{1}$ if $x \neq 0$,

(ii) $\mathrm{inv}\,x = 0$, otherwise.

Let us notice that the functor $\mathrm{inv}\,x$ is involutive.

4. Definition of the Set of All Complex Numbers

In the sequel a, b denote elements of $\mathbb{R}$.

We now state the proposition

(10)[2] $[0 \longmapsto a, \mathbf{1} \longmapsto b] \notin \mathbb{R}$.

Let x, y be elements of $\mathbb{R}$. The functor $x + yi$ yielding an element of $\mathbb{C}$ is defined as follows:

(Def. 7)[3] $x + yi = \begin{cases} \text{(i)} \quad x, \text{ if } y = 0, \\ [0 \longmapsto x, \mathbf{1} \longmapsto y], \text{ otherwise.} \end{cases}$

We now state two propositions:

(11) For every element c of $\mathbb{C}$ there exist elements r, s of $\mathbb{R}$ such that $c = r + si$.

(12) For all elements x_1, x_2, y_1, y_2 of $\mathbb{R}$ such that $x_1 + x_2 i = y_1 + y_2 i$ holds $x_1 = y_1$ and $x_2 = y_2$.

Next we state a number of propositions:

(13) For all elements x, o of $\mathbb{R}$ such that $o = 0$ holds $+(x, o) = x$.

(14) For all elements x, o of $\mathbb{R}$ such that $o = 0$ holds $\cdot(x, o) = 0$.

(15) For all elements x, y, z of $\mathbb{R}$ holds $\cdot(x, \cdot(y, z)) = \cdot(\cdot(x, y), z)$.

(16) For all elements x, y, z of $\mathbb{R}$ holds $\cdot(x, +(y, z)) = +(\cdot(x, y), \cdot(x, z))$.

[2] The proposition (9) has been removed.

[3] The definition (Def. 6) has been removed.

(17) For all elements x, y of $\mathbb{R}$ holds $\cdot({}^{\rm op}x, y) = {}^{\rm op}\cdot(x,y)$.

(18) For every element x of $\mathbb{R}$ holds $\cdot(x,x) \in \mathbb{R}_+$.

(19) For all x, y such that $+(\cdot(x,x), \cdot(y,y)) = 0$ holds $x = 0$.

(20) For all elements x, y, z of $\mathbb{R}$ such that $x \neq 0$ and $\cdot(x,y) = \mathbf{1}$ and $\cdot(x,z) = \mathbf{1}$ holds $y = z$.

(21) For all x, y such that $y = \mathbf{1}$ holds $\cdot(x,y) = x$.

(22) For all x, y such that $y \neq 0$ holds $\cdot(\cdot(x,y), \mathrm{inv}\, y) = x$.

(23) For all x, y such that $\cdot(x,y) = 0$ holds $x = 0$ or $y = 0$.

(24) For all x, y holds $\mathrm{inv}\cdot(x,y) = \cdot(\mathrm{inv}\, x, \mathrm{inv}\, y)$.

(25) For all elements x, y, z of $\mathbb{R}$ holds $+(x, +(y,z)) = +(+(x,y), z)$.

(26) If $x + yi \in \mathbb{R}$, then $y = 0$.

(27) For all elements x, y of $\mathbb{R}$ holds ${}^{\rm op}+(x,y) = +({}^{\rm op}x, {}^{\rm op}y)$.

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