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Inst. of Computer Science, Univ. of Białystok

Ternary Fields[1]

Michał Muzalewski
Warsaw University
Białystok

Wojciech Skaba
Nicolaus Copernicus University
Toruń

Summary. This article contains part 3 of the set of papers concerning the theory of algebraic structures, based on the book [3, pp. 13–15] (pages 6–8 for English edition).

First the basic structure $\langle F, 0, 1, T \rangle$ is defined, where T is a ternary operation on F (three argument operations have been introduced in the article [2]. Following it, the basic axioms of a ternary field are displayed, the mode is defined and its existence proved. The basic properties of a ternary field are also contemplated there.

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The articles [6], [4], [5], [1], and [2] provide the notation and terminology for this paper.

We consider ternary field structures as extensions of zero structure as systems

⟨ a carrier, a zero, a unity, an operation ⟩,

where the carrier is a set, the zero and the unity are elements of the carrier, and the operation is a ternary operation on the carrier.

One can verify that there exists a ternary field structure which is non empty.

In the sequel F denotes a non empty ternary field structure.

Let us consider F. A scalar of F is an element of F.

In the sequel a, b, c denote scalars of F.

Let us consider F, a, b, c. The functor $\mathrm{T}(a,b,c)$ yielding a scalar of F is defined as follows:

(Def. 1) $\mathrm{T}(a,b,c) =$ (the operation of $F)(a, b, c)$.

Let us consider F. The functor $\mathbf{1}_F$ yields a scalar of F and is defined as follows:

(Def. 3)[1] $\mathbf{1}_F =$ the unity of F.

The ternary operation $\mathrm{T}_{\mathbb{R}}$ on $\mathbb{R}$ is defined as follows:

(Def. 4) For all real numbers a, b, c holds $\mathrm{T}_{\mathbb{R}}(a, b, c) = a \cdot b + c$.

The strict ternary field structure $\mathbb{R}_{\mathrm{t}}$ is defined as follows:

(Def. 5) $\mathbb{R}_{\mathrm{t}} = \langle \mathbb{R}, 0, 1, \mathrm{T}_{\mathbb{R}} \rangle$.

Let us observe that $\mathbb{R}_{\mathrm{t}}$ is non empty.

Let a, b, c be scalars of $\mathbb{R}_{\mathrm{t}}$. The functor $\mathrm{T}^e(a,b,c)$ yields a scalar of $\mathbb{R}_{\mathrm{t}}$ and is defined as follows:

(Def. 6) $\mathrm{T}^e(a,b,c) =$ (the operation of $\mathbb{R}_{\mathrm{t}})(a, b, c)$.

Next we state four propositions:

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[1] The definition (Def. 2) has been removed.

(3)[2] For all real numbers u, u', v, v' such that $u \neq u'$ there exists a real number x such that $u \cdot x + v = u' \cdot x + v'$.

(5)[3] For all scalars u, a, v of $\mathbb{R}_{\rm t}$ and for all real numbers z, x, y such that $u = z$ and $a = x$ and $v = y$ holds $\mathrm{T}(u, a, v) = z \cdot x + y$.

(6) $0 = 0_{\mathbb{R}_{\rm t}}$.

(7) $1 = \mathbf{1}_{\mathbb{R}_{\rm t}}$.

Let I_1 be a non empty ternary field structure. We say that I_1 is ternary field-like if and only if the conditions (Def. 7) are satisfied.

(Def. 7) $0_{(I_1)} \neq \mathbf{1}_{(I_1)}$ and for every scalar a of I_1 holds $\mathrm{T}(a, \mathbf{1}_{(I_1)}, 0_{(I_1)}) = a$ and for every scalar a of I_1 holds $\mathrm{T}(\mathbf{1}_{(I_1)}, a, 0_{(I_1)}) = a$ and for all scalars a, b of I_1 holds $\mathrm{T}(a, 0_{(I_1)}, b) = b$ and for all scalars a, b of I_1 holds $\mathrm{T}(0_{(I_1)}, a, b) = b$ and for all scalars u, a, b of I_1 there exists a scalar v of I_1 such that $\mathrm{T}(u, a, v) = b$ and for all scalars u, a, v, v' of I_1 such that $\mathrm{T}(u, a, v) = \mathrm{T}(u, a, v')$ holds $v = v'$ and for all scalars a, a' of I_1 such that $a \neq a'$ and for all scalars b, b' of I_1 there exist scalars u, v of I_1 such that $\mathrm{T}(u, a, v) = b$ and $\mathrm{T}(u, a', v) = b'$ and for all scalars u, u' of I_1 such that $u \neq u'$ and for all scalars v, v' of I_1 there exists a scalar a of I_1 such that $\mathrm{T}(u, a, v) = \mathrm{T}(u', a, v')$ and for all scalars a, a', u, u', v, v' of I_1 such that $\mathrm{T}(u, a, v) = \mathrm{T}(u', a, v')$ and $\mathrm{T}(u, a', v) = \mathrm{T}(u', a', v')$ holds $a = a'$ or $u = u'$.

Let us observe that there exists a non empty ternary field structure which is strict and ternary field-like.

A ternary field is a ternary field-like non empty ternary field structure.

We adopt the following rules: F denotes a ternary field and a, a', b, c, x, x', u, u', v, v' denote scalars of F.

Next we state several propositions:

(8) If $a \neq a'$ and $\mathrm{T}(u, a, v) = \mathrm{T}(u', a, v')$ and $\mathrm{T}(u, a', v) = \mathrm{T}(u', a', v')$, then $u = u'$ and $v = v'$.

(11)[4] If $a \neq 0_F$, then for all b, c there exists x such that $\mathrm{T}(a, x, b) = c$.

(12) If $a \neq 0_F$ and $\mathrm{T}(a, x, b) = \mathrm{T}(a, x', b)$, then $x = x'$.

(13) If $a \neq 0_F$, then for all b, c there exists x such that $\mathrm{T}(x, a, b) = c$.

(14) If $a \neq 0_F$ and $\mathrm{T}(x, a, b) = \mathrm{T}(x', a, b)$, then $x = x'$.

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[5] Wojciech A. Trybulec. Vectors in real linear space. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/rlvect_1.html`.

[2] The propositions (1) and (2) have been removed.
[3] The proposition (4) has been removed.
[4] The propositions (9) and (10) have been removed.

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