JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

# One-Dimensional Congruence of Segments, Basic Facts and Midpoint Relation[1]

Barbara Konstanta
Siedlce Agricultural
and Pedagogical University

Urszula Kowieska
Siedlce Agricultural
and Pedagogical University

Grzegorz Lewandowski
Siedlce Agricultural
and Pedagogical University

Krzysztof Prażmowski
Warsaw University
Białystok

**Summary.** We study a theory of one-dimensional congruence of segments. The theory is characterized by a suitable formal axiom system; as a model of this system one can take the structure obtained from any weak directed geometrical bundle, with the congruence interpreted as in the case of "classical" vectors. Preliminary consequences of our axiom system are proved, basic relations of maximal distance and of midpoint are defined, and several fundamental properties of them are established.

MML Identifier: AFVECT01.

WWW: http://mizar.org/JFM/Vol2/afvect01.html

The articles [4], [2], [5], [3], [6], and [1] provide the notation and terminology for this paper.

Let $A$ be a non empty set and let $C$ be a binary relation on $[:A,A:]$. Observe that $\langle A,C\rangle$ is non empty.

Let $I_1$ be a non empty affine structure. We say that $I_1$ is weak segment-congruence space-like if and only if the conditions (Def. 2) are satisfied.

(Def. 2)[1] For all elements $a$, $b$ of $I_1$ holds $a,b \upharpoonright\!\upharpoonright b,a$ and for all elements $a$, $b$ of $I_1$ such that $a,b \upharpoonright\!\upharpoonright a,a$ holds $a=b$ and for all elements $a$, $b$, $c$, $d$, $p$, $q$ of $I_1$ such that $a,b \upharpoonright\!\upharpoonright p,q$ and $c,d \upharpoonright\!\upharpoonright p,q$ holds $a,b \upharpoonright\!\upharpoonright c,d$ and for all elements $a$, $c$ of $I_1$ there exists an element $b$ of the carrier of $I_1$ such that $a,b \upharpoonright\!\upharpoonright b,c$ and for all elements $a$, $a'$, $b$, $b'$, $p$ of $I_1$ such that $a \neq a'$ and $b \neq b'$ and $p,a \upharpoonright\!\upharpoonright p,a'$ and $p,b \upharpoonright\!\upharpoonright p,b'$ holds $a,b \upharpoonright\!\upharpoonright a',b'$ and for all elements $a$, $b$ of $I_1$ holds $a=b$ or there exists an element $c$ of $I_1$ such that $a \neq c$ and $a,b \upharpoonright\!\upharpoonright b,c$ or there exist elements $p$, $p'$ of $I_1$ such that $p \neq p'$ and $a,b \upharpoonright\!\upharpoonright p,p'$ and $a,p \upharpoonright\!\upharpoonright p,b$ and $a,p' \upharpoonright\!\upharpoonright p',b$ and for all elements $a$, $b$, $b'$, $p$, $p'$, $c$ of $I_1$ such that $a,b \upharpoonright\!\upharpoonright b,c$ and $b,b' \upharpoonright\!\upharpoonright p,p'$ and $b,p \upharpoonright\!\upharpoonright p,b'$ and $b,p' \upharpoonright\!\upharpoonright p',b'$ holds $a,b' \upharpoonright\!\upharpoonright b',c$ and for all elements $a$, $b$, $b'$, $c$ of $I_1$ such that $a \neq c$ and $b \neq b'$ and $a,b \upharpoonright\!\upharpoonright b,c$ and $a,b' \upharpoonright\!\upharpoonright b',c$ there exist elements $p$, $p'$ of $I_1$ such that $p \neq p'$ and $b,b' \upharpoonright\!\upharpoonright p,p'$ and $b,p \upharpoonright\!\upharpoonright p,b'$ and $b,p' \upharpoonright\!\upharpoonright p',b'$ and for all elements $a$, $b$, $c$, $p$, $p'$, $q$, $q'$ of $I_1$ such that $a,b \upharpoonright\!\upharpoonright p,p'$ and $a,c \upharpoonright\!\upharpoonright q,q'$ and $a,p \upharpoonright\!\upharpoonright p,b$ and $a,q \upharpoonright\!\upharpoonright q,c$ and $a,p' \upharpoonright\!\upharpoonright p',b$ and $a,q' \upharpoonright\!\upharpoonright q',c$ there exist elements $r$, $r'$ of $I_1$ such that $b,c \upharpoonright\!\upharpoonright r,r'$ and $b,r \upharpoonright\!\upharpoonright r,c$ and $b,r' \upharpoonright\!\upharpoonright r',c$.

One can verify that there exists a non empty affine structure which is strict, non trivial, and weak segment-congruence space-like.

[1] Supported by RPBP.III-24.C3.

[1] The definition (Def. 1) has been removed.



© Association of Mizar Users

A weak segment-congruence space is a non trivial weak segment-congruence space-like non empty affine structure.

We adopt the following rules: $A_1$ is a weak segment-congruence space and $a$, $b$, $b'$, $b''$, $c$, $d$, $p$, $p'$ are elements of $A_1$.

The following propositions are true:

(1) $a,b \upharpoonright\!\upharpoonright a,b$.

(2) If $a,b \upharpoonright\!\upharpoonright c,d$, then $c,d \upharpoonright\!\upharpoonright a,b$.

(3) If $a,b \upharpoonright\!\upharpoonright c,d$, then $a,b \upharpoonright\!\upharpoonright d,c$.

(4) If $a,b \upharpoonright\!\upharpoonright c,d$, then $b,a \upharpoonright\!\upharpoonright c,d$.

(5) For all $a$, $b$ holds $a,a \upharpoonright\!\upharpoonright b,b$.

(6) If $a,b \upharpoonright\!\upharpoonright c,c$, then $a = b$.

(7) If $a,b \upharpoonright\!\upharpoonright p,p'$ and $a,b \upharpoonright\!\upharpoonright b,c$ and $a,p \upharpoonright\!\upharpoonright p,b$ and $a,p' \upharpoonright\!\upharpoonright p',b$, then $a = c$.

(8) If $a,b' \upharpoonright\!\upharpoonright a,b''$ and $a,b \upharpoonright\!\upharpoonright a,b''$, then $b = b'$ or $b = b''$ or $b' = b''$.

Let us consider $A_1$ and let us consider $a$, $b$. We say that $a$, $b$ are in a maximal distance if and only if:

(Def. 4)[2] There exist $p$, $p'$ such that $p \neq p'$ and $a,b \upharpoonright\!\upharpoonright p,p'$ and $a,p \upharpoonright\!\upharpoonright p,b$ and $a,p' \upharpoonright\!\upharpoonright p',b$.

Let us consider $A_1$ and let us consider $a$, $b$, $c$. We say that $b$ is a midpoint of $a$, $c$ if and only if:

(Def. 5) $a = b$ and $b = c$ and $a = c$ or $a = c$ and $a$, $b$ are in a maximal distance or $a \neq c$ and $a,b \upharpoonright\!\upharpoonright b,c$.

Next we state three propositions:

(11)[3] If $a \neq b$ and $a$, $b$ are not in a maximal distance, then there exists $c$ such that $a \neq c$ and $a,b \upharpoonright\!\upharpoonright b,c$.

(12) If $a$, $b$ are in a maximal distance and $a,b \upharpoonright\!\upharpoonright b,c$, then $a = c$.

(13) If $a$, $b$ are in a maximal distance, then $a \neq b$.

## REFERENCES


[1] Józef Białas. Group and field definitions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/realset1.html.

[2] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[3] Henryk Oryszczyszyn and Krzysztof Prażmowski. Analytical ordered affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/analoaf.html.

[4] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[5] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[6] Edmund Woronowicz. Relations defined on sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relset_1.html.


*Received October 4, 1990*

*Published January 2, 2004*

[2] The definition (Def. 3) has been removed.

[3] The propositions (9) and (10) have been removed.