JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

Classical Configurations in Affine Planes[1]

Henryk Oryszczyszyn
Warsaw University
Białystok

Krzysztof Prażmowski
Warsaw University
Białystok

Summary. The classical sequence of implications which hold between Desargues and Pappus Axioms is proved. Formally Minor and Major Desargues Axiom (as suitable properties – predicates – of an affine plane) together with all its indirect forms are introduced; the same procedure is applied to Pappus Axioms. The so called Trapezium Desargues Axiom is also considered.

MML Identifier: AFF_2.

WWW: http://mizar.org/JFM/Vol2/aff_2.html

The articles [1], [2], and [3] provide the notation and terminology for this paper.

We adopt the following rules: A_1 is an affine plane, a, a', b, b', c, c', o are elements of A_1, and A, C, K, M, N, P are subsets of A_1.

Let us consider A_1. We say that A_1 satisfies **PPAP** if and only if the condition (Def. 1) is satisfied.

(Def. 1) Let given M, N, a, b, c, a', b', c'. Suppose M is a line and N is a line and $a \in M$ and $b \in M$ and $c \in M$ and $a' \in N$ and $b' \in N$ and $c' \in N$ and $a,b' \upuparrows b,a'$ and $b,c' \upuparrows c,b'$. Then $a,c' \upuparrows c,a'$.

We introduce A_1 satisfies **PPAP** as a synonym of A_1 satisfies **PPAP**.

Let A_1 be an affine space. We say that A_1 is Pappian if and only if the condition (Def. 2) is satisfied.

(Def. 2) Let M, N be subsets of A_1 and o, a, b, c, a', b', c' be elements of A_1. Suppose that M is a line and N is a line and $M \neq N$ and $o \in M$ and $o \in N$ and $o \neq a$ and $o \neq a'$ and $o \neq b$ and $o \neq b'$ and $o \neq c$ and $o \neq c'$ and $a \in M$ and $b \in M$ and $c \in M$ and $a' \in N$ and $b' \in N$ and $c' \in N$ and $a,b' \upuparrows b,a'$ and $b,c' \upuparrows c,b'$. Then $a,c' \upuparrows c,a'$.

We introduce A_1 satisfies **PAP** as a synonym of A_1 is Pappian.

Let us consider A_1. We say that A_1 satisfies **PAP**$_1$ if and only if the condition (Def. 3) is satisfied.

(Def. 3) Let given M, N, o, a, b, c, a', b', c'. Suppose that M is a line and N is a line and $M \neq N$ and $o \in M$ and $o \in N$ and $o \neq a$ and $o \neq a'$ and $o \neq b$ and $o \neq b'$ and $o \neq c$ and $o \neq c'$ and $a \in M$ and $b \in M$ and $c \in M$ and $b' \in N$ and $c' \in N$ and $a,b' \upuparrows b,a'$ and $b,c' \upuparrows c,b'$ and $a,c' \upuparrows c,a'$ and $b \neq c$. Then $a' \in N$.

We introduce A_1 satisfies **PAP**$_1$ as a synonym of A_1 satisfies **PAP**$_1$.

Let A_1 be an affine space. We say that A_1 is Desarguesian if and only if the condition (Def. 4) is satisfied.

(Def. 4) Let A, P, C be subsets of A_1 and o, a, b, c, a', b', c' be elements of A_1. Suppose that $o \in A$ and $o \in P$ and $o \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$. Then $b,c \upuparrows b',c'$.

[1]Supported by RPBP.III-24.C2.

We introduce A_1 satisfies **DES** as a synonym of A_1 is Desarguesian.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES}_1$ if and only if the condition (Def. 5) is satisfied.

(Def. 5) Let given A, P, C, o, a, b, c, a', b', c'. Suppose that $o \in A$ and $o \in P$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and $b,c \upuparrows b',c'$ and not $\mathbf{L}(a,b,c)$ and $c \neq c'$. Then $o \in C$.

We introduce A_1 satisfies $\mathbf{DES}_1$ as a synonym of A_1 satisfies $\mathbf{DES}_1$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES}_2$ if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let given A, P, C, o, a, b, c, a', b', c'. Suppose that $o \in A$ and $o \in P$ and $o \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and $b,c \upuparrows b',c'$. Then $c' \in C$.

We introduce A_1 satisfies $\mathbf{DES}_2$ as a synonym of A_1 satisfies $\mathbf{DES}_2$.

Let A_1 be an affine space. We say that A_1 is Moufangian if and only if the condition (Def. 7) is satisfied.

(Def. 7) Let K be a subset of A_1 and o, a, b, c, a', b', c' be elements of A_1. Suppose K is a line and $o \in K$ and $c \in K$ and $c' \in K$ and $a \notin K$ and $o \neq c$ and $a \neq b$ and $\mathbf{L}(o,a,a')$ and $\mathbf{L}(o,b,b')$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and a, b // K. Then $b,c \upuparrows b',c'$.

We introduce A_1 satisfies **TDES** as a synonym of A_1 is Moufangian.

Let us consider A_1. We say that A_1 satisfies $\mathbf{TDES}_1$ if and only if the condition (Def. 8) is satisfied.

(Def. 8) Let given K, o, a, b, c, a', b', c'. Suppose K is a line and $o \in K$ and $c \in K$ and $c' \in K$ and $a \notin K$ and $o \neq c$ and $a \neq b$ and $\mathbf{L}(o,a,a')$ and $a,b \upuparrows a',b'$ and $b,c \upuparrows b',c'$ and $a,c \upuparrows a',c'$ and a, b // K. Then $\mathbf{L}(o,b,b')$.

We introduce A_1 satisfies $\mathbf{TDES}_1$ as a synonym of A_1 satisfies $\mathbf{TDES}_1$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{TDES}_2$ if and only if the condition (Def. 9) is satisfied.

(Def. 9) Let given K, o, a, b, c, a', b', c'. Suppose K is a line and $o \in K$ and $c \in K$ and $c' \in K$ and $a \notin K$ and $o \neq c$ and $a \neq b$ and $\mathbf{L}(o,a,a')$ and $\mathbf{L}(o,b,b')$ and $b,c \upuparrows b',c'$ and $a,c \upuparrows a',c'$ and a, b // K. Then $a,b \upuparrows a',b'$.

We introduce A_1 satisfies $\mathbf{TDES}_2$ as a synonym of A_1 satisfies $\mathbf{TDES}_2$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{TDES}_3$ if and only if the condition (Def. 10) is satisfied.

(Def. 10) Let given K, o, a, b, c, a', b', c'. Suppose K is a line and $o \in K$ and $c \in K$ and $a \notin K$ and $o \neq c$ and $a \neq b$ and $\mathbf{L}(o,a,a')$ and $\mathbf{L}(o,b,b')$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and $b,c \upuparrows b',c'$ and a, b // K. Then $c' \in K$.

We introduce A_1 satisfies $\mathbf{TDES}_3$ as a synonym of A_1 satisfies $\mathbf{TDES}_3$.

Let A_1 be an affine space. We say that A_1 is translational if and only if the condition (Def. 11) is satisfied.

(Def. 11) Let A, P, C be subsets of A_1 and a, b, c, a', b', c' be elements of A_1. Suppose that A // P and A // C and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$. Then $b,c \upuparrows b',c'$.

We introduce A_1 satisfies **des** as a synonym of A_1 is translational.

Let us consider A_1. We say that A_1 satisfies $\mathbf{des}_1$ if and only if the condition (Def. 12) is satisfied.

(Def. 12) Let given A, P, C, a, b, c, a', b', c'. Suppose that A // P and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and $b,c \upuparrows b',c'$ and not $\mathbf{L}(a,b,c)$ and $c \neq c'$. Then A // C.

We introduce A_1 satisfies $\mathbf{des}_1$ as a synonym of A_1 satisfies $\mathbf{des}_1$.

Let A_1 be an affine space. We say that A_1 satisfies pap if and only if the condition (Def. 13) is satisfied.

(Def. 13) Let M, N be subsets of A_1 and a, b, c, a', b', c' be elements of A_1. Suppose M is a line and N is a line and $a \in M$ and $b \in M$ and $c \in M$ and $M \parallel N$ and $M \neq N$ and $a' \in N$ and $b' \in N$ and $c' \in N$ and $a, b' \upuparrows b, a'$ and $b, c' \upuparrows c, b'$. Then $a, c' \upuparrows c, a'$.

We introduce A_1 satisfies **pap** as a synonym of A_1 satisfies pap.

Let us consider A_1. We say that A_1 satisfies $\mathbf{pap}_1$ if and only if the condition (Def. 14) is satisfied.

(Def. 14) Let given M, N, a, b, c, a', b', c'. Suppose that M is a line and N is a line and $a \in M$ and $b \in M$ and $c \in M$ and $M \parallel N$ and $M \neq N$ and $a' \in N$ and $b' \in N$ and $a, b' \upuparrows b, a'$ and $b, c' \upuparrows c, b'$ and $a, c' \upuparrows c, a'$ and $a' \neq b'$. Then $c' \in N$.

We introduce A_1 satisfies $\mathbf{pap}_1$ as a synonym of A_1 satisfies $\mathbf{pap}_1$.

The following propositions are true:

(15)[1] A_1 satisfies **PAP** iff A_1 satisfies $\mathbf{PAP}_1$.

(16) A_1 satisfies **DES** iff A_1 satisfies $\mathbf{DES}_1$.

(17) If A_1 satisfies **TDES**, then A_1 satisfies $\mathbf{TDES}_1$.

(18) If A_1 satisfies $\mathbf{TDES}_1$, then A_1 satisfies $\mathbf{TDES}_2$.

(19) If A_1 satisfies $\mathbf{TDES}_2$, then A_1 satisfies $\mathbf{TDES}_3$.

(20) If A_1 satisfies $\mathbf{TDES}_3$, then A_1 satisfies **TDES**.

(21) A_1 satisfies **des** iff A_1 satisfies $\mathbf{des}_1$.

(22) A_1 satisfies **pap** iff A_1 satisfies $\mathbf{pap}_1$.

(23) If A_1 satisfies **PAP**, then A_1 satisfies **pap**.

(24) A_1 satisfies **PPAP** iff A_1 satisfies **PAP** and A_1 satisfies **pap**.

(25) If A_1 satisfies **PAP**, then A_1 satisfies **DES**.

(26) If A_1 satisfies **DES**, then A_1 satisfies **TDES**.

(27) If A_1 satisfies $\mathbf{TDES}_1$, then A_1 satisfies $\mathbf{des}_1$.

(28) If A_1 satisfies **TDES**, then A_1 satisfies **des**.

(29) If A_1 satisfies **des**, then A_1 satisfies **pap**.

References

[1] Henryk Oryszczyszyn and Krzysztof Prażmowski. Analytical ordered affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/analoaf.html.

[2] Henryk Oryszczyszyn and Krzysztof Prażmowski. Ordered affine spaces defined in terms of directed parallelity — part I. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/diraf.html.

[1] The propositions (1)–(14) have been removed.

[3] Henryk Oryszczyszyn and Krzysztof Prażmowski. Parallelity and lines in affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/aff_1.html.

Received April 13, 1990

Published January 2, 2004