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The Contraction Lemma

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Summary. The article includes the proof of the contraction lemma which claims that every class in which the axiom of extensionality is valid is isomorphic with a transitive class. In this article the isomorphism (wrt membership relation) of two sets is defined. It is based on [6].

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The articles [7], [8], [9], [4], [1], [5], [3], and [2] provide the notation and terminology for this paper.

For simplicity, we follow the rules: X, Y, Z are sets, x, y are sets, E is a non empty set, A, B, C are ordinal numbers, L is a transfinite sequence, f is a function, and d, d_1, d' are elements of E.

Let us consider E, A. The functor E_A yields a set and is defined by the condition (Def. 1).

(Def. 1) There exists L such that

(i) $E_A = \{d : \bigwedge_{d_1} (d_1 \in d \Rightarrow \bigvee_B (B \in \mathrm{dom} L \wedge d_1 \in \bigcup\{L(B)\}))\}$,

(ii) $\mathrm{dom} L = A$, and

(iii) for every B such that $B \in A$ holds $L(B) = \{d_1 : \bigwedge_d (d \in d_1 \Rightarrow \bigvee_C (C \in \mathrm{dom}(L \restriction B) \wedge d \in \bigcup\{(L \restriction B)(C)\}))\}$.

One can prove the following propositions:

(1) $E_A = \{d : \bigwedge_{d_1} (d_1 \in d \Rightarrow \bigvee_B (B \in A \wedge d_1 \in E_B))\}$.

(2) It is not true that there exists d_1 such that $d_1 \in d$ iff $d \in E_\emptyset$.

(3) $d \cap E \subseteq E_A$ iff $d \in E_{\mathrm{succ} A}$.

(4) If $A \subseteq B$, then $E_A \subseteq E_B$.

(5) There exists A such that $d \in E_A$.

(6) If $d' \in d$ and $d \in E_A$, then $d' \in E_A$ and there exists B such that $B \in A$ and $d' \in E_B$.

(7) $E_A \subseteq E$.

(8) There exists A such that $E = E_A$.

(9) There exists f such that $\mathrm{dom} f = E$ and for every d holds $f(d) = f^\circ d$.

Let us consider f, X, Y. We say that f is an isomorphism between X and Y if and only if the conditions (Def. 2) are satisfied.

(Def. 2)(i) dom $f = X$,

(ii) rng $f = Y$,

(iii) f is one-to-one, and

(iv) for all x, y such that $x \in X$ and $y \in X$ holds there exists Z such that $Z = y$ and $x \in Z$ iff there exists Z such that $f(y) = Z$ and $f(x) \in Z$.

Let us consider X, Y. We say that X and Y are isomorphic if and only if:

(Def. 3) There exists f which is an isomorphism between X and Y.

Next we state the proposition

(12)[1] If dom $f = E$ and for every d holds $f(d) = f^{\circ}d$, then rng f is transitive.

In the sequel u, v, w denote elements of E.
Next we state two propositions:

(13) If $E \models$ the axiom of extensionality, then for all u, v such that for every w holds $w \in u$ iff $w \in v$ holds $u = v$.

(14) If $E \models$ the axiom of extensionality, then there exists X such that X is transitive and E and X are isomorphic.

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[1] The propositions (10) and (11) have been removed.