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Miscellaneous Facts about Functors

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Summary. In the paper we show useful facts concerning reverse and inclusion functors and the restriction of functors. We also introduce a new notation for the intersection of categories and the isomorphism under arbitrary functors.

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The articles [13], [7], [20], [21], [22], [4], [5], [14], [3], [12], [8], [6], [15], [16], [11], [10], [2], [17], [18], [19], [9], and [1] provide the notation and terminology for this paper.

1. Reverse Functors

One can prove the following propositions:

(1) Let A, B be transitive non empty category structures with units and F be a feasible reflexive functor structure from A to B. Suppose F is coreflexive and bijective. Let a be an object of A and b be an object of B. Then $F(a) = b$ if and only if $F^{-1}(b) = a$.

(2) Let A, B be transitive non empty category structures with units, F be a precovariant feasible functor structure from A to B, and G be a precovariant feasible functor structure from B to A. Suppose F is bijective and $G = F^{-1}$. Let a_1, a_2 be objects of A. Suppose $\langle a_1, a_2\rangle \neq \emptyset$. Let f be a morphism from a_1 to a_2 and g be a morphism from $F(a_1)$ to $F(a_2)$. Then $F(f) = g$ if and only if $G(g) = f$.

(3) Let A, B be transitive non empty category structures with units, F be a precontravariant feasible functor structure from A to B, and G be a precontravariant feasible functor structure from B to A. Suppose F is bijective and $G = F^{-1}$. Let a_1, a_2 be objects of A. Suppose $\langle a_1, a_2\rangle \neq \emptyset$. Let f be a morphism from a_1 to a_2 and g be a morphism from $F(a_2)$ to $F(a_1)$. Then $F(f) = g$ if and only if $G(g) = f$.

(4) Let A, B be categories and F be a functor from A to B. Suppose F is bijective. Let G be a functor from B to A. If $F \cdot G = \mathrm{id}_B$, then the functor structure of $G = F^{-1}$.

(5) Let A, B be categories and F be a functor from A to B. Suppose F is bijective. Let G be a functor from B to A. If $G \cdot F = \mathrm{id}_A$, then the functor structure of $G = F^{-1}$.

(6) Let A, B be categories and F be a covariant functor from A to B. Suppose F is bijective. Let G be a covariant functor from B to A. Suppose that

(i) for every object b of B holds $F(G(b)) = b$, and

(ii) for all objects a, b of B such that $\langle a, b\rangle \neq \emptyset$ and for every morphism f from a to b holds $F(G(f)) = f$.

Then the functor structure of $G = F^{-1}$.

(7) Let A, B be categories and F be a contravariant functor from A to B. Suppose F is bijective. Let G be a contravariant functor from B to A. Suppose that

(i) for every object b of B holds $F(G(b)) = b$, and

(ii) for all objects a, b of B such that $\langle a, b \rangle \neq \emptyset$ and for every morphism f from a to b holds $F(G(f)) = f$.

Then the functor structure of $G = F^{-1}$.

(8) Let A, B be categories and F be a covariant functor from A to B. Suppose F is bijective. Let G be a covariant functor from B to A. Suppose that

(i) for every object a of A holds $G(F(a)) = a$, and

(ii) for all objects a, b of A such that $\langle a, b \rangle \neq \emptyset$ and for every morphism f from a to b holds $G(F(f)) = f$.

Then the functor structure of $G = F^{-1}$.

(9) Let A, B be categories and F be a contravariant functor from A to B. Suppose F is bijective. Let G be a contravariant functor from B to A. Suppose that

(i) for every object a of A holds $G(F(a)) = a$, and

(ii) for all objects a, b of A such that $\langle a, b \rangle \neq \emptyset$ and for every morphism f from a to b holds $G(F(f)) = f$.

Then the functor structure of $G = F^{-1}$.

2. Intersection of Categories

Let A, B be category structures. We say that A and B have the same composition if and only if:

(Def. 1) For all sets a_1, a_2, a_3 holds (the composition of A)($\langle a_1, a_2, a_3 \rangle$) $\approx$ (the composition of B)($\langle a_1, a_2, a_3 \rangle$).

Let us note that the predicate A and B have the same composition is symmetric.

Next we state three propositions:

(10) Let A, B be category structures. Then A and B have the same composition if and only if for all sets a_1, a_2, a_3, x such that $x \in$ dom(the composition of A)($\langle a_1, a_2, a_3 \rangle$) and $x \in$ dom(the composition of B)($\langle a_1, a_2, a_3 \rangle$) holds (the composition of A)($\langle a_1, a_2, a_3 \rangle$)($x$) = (the composition of B)($\langle a_1, a_2, a_3 \rangle$)($x$).

(11) Let A, B be transitive non empty category structures. Then A and B have the same composition if and only if for all objects a_1, a_2, a_3 of A such that $\langle a_1, a_2 \rangle \neq \emptyset$ and $\langle a_2, a_3 \rangle \neq \emptyset$ and for all objects b_1, b_2, b_3 of B such that $\langle b_1, b_2 \rangle \neq \emptyset$ and $\langle b_2, b_3 \rangle \neq \emptyset$ and $b_1 = a_1$ and $b_2 = a_2$ and $b_3 = a_3$ and for every morphism f_1 from a_1 to a_2 and for every morphism g_1 from b_1 to b_2 such that $g_1 = f_1$ and for every morphism f_2 from a_2 to a_3 and for every morphism g_2 from b_2 to b_3 such that $g_2 = f_2$ holds $f_2 \cdot f_1 = g_2 \cdot g_1$.

(12) For all para-functional semi-functional categories A, B holds A and B have the same composition.

Let f, g be functions. The functor Intersect(f, g) yields a function and is defined as follows:

(Def. 2) dom Intersect$(f, g) = \operatorname{dom} f \cap \operatorname{dom} g$ and for every set x such that $x \in \operatorname{dom} f \cap \operatorname{dom} g$ holds (Intersect(f, g))$(x) = f(x) \cap g(x)$.

Let us notice that the functor Intersect(f, g) is commutative.

One can prove the following propositions:

(13) For every set I and for all many sorted sets A, B indexed by I holds Intersect$(A, B) = A \cap B$.

(14) Let I, J be sets, A be a many sorted set indexed by I, and B be a many sorted set indexed by J. Then Intersect(A,B) is a many sorted set indexed by $I \cap J$.

(15) Let I, J be sets, A be a many sorted set indexed by I, B be a function, and C be a many sorted set indexed by J. If $C = \text{Intersect}(A,B)$, then $C \dot{\subseteq} A$.

(16) Let A_1, A_2, B_1, B_2 be sets, f be a function from A_1 into A_2, and g be a function from B_1 into B_2. If $f \approx g$, then $f \cap g$ is a function from $A_1 \cap B_1$ into $A_2 \cap B_2$.

(17) Let I_1, I_2 be sets, A_1, B_1 be many sorted sets indexed by I_1, A_2, B_2 be many sorted sets indexed by I_2, and A, B be many sorted sets indexed by $I_1 \cap I_2$. Suppose $A = \text{Intersect}(A_1,A_2)$ and $B = \text{Intersect}(B_1,B_2)$. Let F be a many sorted function from A_1 into B_1 and G be a many sorted function from A_2 into B_2. Suppose that for every set x such that $x \in \text{dom} F$ and $x \in \text{dom} G$ holds $F(x) \approx G(x)$. Then Intersect(F,G) is a many sorted function from A into B.

(18) Let I, J be sets, F be a many sorted set indexed by $[: I, I :]$, and G be a many sorted set indexed by $[: J, J :]$. Then there exists a many sorted set H indexed by $[: I \cap J, I \cap J :]$ such that $H = \text{Intersect}(F,G)$ and $\text{Intersect}(\{|F|\}, \{|G|\}) = \{|H|\}$.

(19) Let I, J be sets, F_1, F_2 be many sorted sets indexed by $[: I, I :]$, and G_1, G_2 be many sorted sets indexed by $[: J, J :]$. Then there exist many sorted sets H_1, H_2 indexed by $[: I \cap J, I \cap J :]$ such that $H_1 = \text{Intersect}(F_1,G_1)$ and $H_2 = \text{Intersect}(F_2,G_2)$ and $\text{Intersect}(\{|F_1,F_2|\}, \{|G_1,G_2|\}) = \{|H_1,H_2|\}$.

Let A, B be category structures. Let us assume that A and B have the same composition. The functor Intersect(A,B) yields a strict category structure and is defined by the conditions (Def. 3).

(Def. 3)(i) The carrier of Intersect(A,B) = (the carrier of A) $\cap$ (the carrier of B),

(ii) the arrows of Intersect(A,B) = Intersect(the arrows of A, the arrows of B), and

(iii) the composition of Intersect(A,B) = Intersect(the composition of A, the composition of B).

Next we state several propositions:

(20) For all category structures A, B such that A and B have the same composition holds Intersect(A,B) = Intersect(B,A).

(21) Let A, B be category structures. Suppose A and B have the same composition. Then Intersect(A,B) is a substructure of A.

(22) Let A, B be category structures. Suppose A and B have the same composition. Let a_1, a_2 be objects of A, b_1, b_2 be objects of B, and o_1, o_2 be objects of Intersect(A,B). If $o_1 = a_1$ and $o_1 = b_1$ and $o_2 = a_2$ and $o_2 = b_2$, then $\langle o_1,o_2 \rangle = (\langle a_1,a_2 \rangle) \cap (\langle b_1,b_2 \rangle)$.

(23) Let A, B be transitive category structures. If A and B have the same composition, then Intersect(A,B) is transitive.

(24) Let A, B be category structures. Suppose A and B have the same composition. Let a_1, a_2 be objects of A, b_1, b_2 be objects of B, and o_1, o_2 be objects of Intersect(A,B). Suppose $o_1 = a_1$ and $o_1 = b_1$ and $o_2 = a_2$ and $o_2 = b_2$ and $\langle a_1,a_2 \rangle \neq \emptyset$ and $\langle b_1,b_2 \rangle \neq \emptyset$. Let f be a morphism from a_1 to a_2 and g be a morphism from b_1 to b_2. If $f = g$, then $f \in \langle o_1,o_2 \rangle$.

(25) Let A, B be non empty category structures with units. Suppose A and B have the same composition. Let a be an object of A, b be an object of B, and o be an object of Intersect(A,B). If $o = a$ and $o = b$ and $\text{id}_a = \text{id}_b$, then $\text{id}_a \in \langle o,o \rangle$.

(26) Let A, B be categories. Suppose that

(i) A and B have the same composition,

(ii) Intersect(A,B) is non empty, and

(iii) for every object a of A and for every object b of B such that $a = b$ holds $\text{id}_a = \text{id}_b$.

Then Intersect(A,B) is a subcategory of A.

3. SUBCATEGORIES

The scheme *SubcategoryUniq* deals with a category $\mathcal{A}$, non empty subcategories $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, a unary predicate $\mathcal{P}$, and a ternary predicate $\mathcal{Q}$, and states that:

The category structure of $\mathcal{B}$ = the category structure of $\mathcal{C}$

provided the parameters have the following properties:

- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{B}$ iff $\mathcal{P}[a]$,
- Let a, b be objects of $\mathcal{A}$ and a', b' be objects of $\mathcal{B}$. Suppose $a' = a$ and $b' = b$ and $\langle a,b\rangle \neq \emptyset$. Let f be a morphism from a to b. Then $f \in \langle a',b'\rangle$ if and only if $\mathcal{Q}[a,b,f]$,
- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{C}$ iff $\mathcal{P}[a]$, and
- Let a, b be objects of $\mathcal{A}$ and a', b' be objects of $\mathcal{C}$. Suppose $a' = a$ and $b' = b$ and $\langle a,b\rangle \neq \emptyset$. Let f be a morphism from a to b. Then $f \in \langle a',b'\rangle$ if and only if $\mathcal{Q}[a,b,f]$.

The following proposition is true

(27) Let A be a non empty category structure and B be a non empty substructure of A. Then B is full if and only if for all objects a_1, a_2 of A and for all objects b_1, b_2 of B such that $b_1 = a_1$ and $b_2 = a_2$ holds $\langle b_1,b_2\rangle = \langle a_1,a_2\rangle$.

Now we present two schemes. The scheme *FullSubcategoryEx* deals with a category $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a strict full non empty subcategory B of $\mathcal{A}$ such that for every object a of $\mathcal{A}$ holds a is an object of B if and only if $\mathcal{P}[a]$

provided the following requirement is met:

- There exists an object a of $\mathcal{A}$ such that $\mathcal{P}[a]$.

The scheme *FullSubcategoryUniq* deals with a category $\mathcal{A}$, full non empty subcategories $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

The category structure of $\mathcal{B}$ = the category structure of $\mathcal{C}$

provided the following requirements are met:

- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{B}$ iff $\mathcal{P}[a]$, and
- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{C}$ iff $\mathcal{P}[a]$.

4. INCLUSION FUNCTORS AND FUNCTOR RESTRICTIONS

Let f be a function yielding function and let x, y be sets. Observe that $f(x,y)$ is relation-like and function-like.

One can prove the following proposition

(28) Let A be a category, C be a non empty subcategory of A, and a, b be objects of C. If $\langle a,b\rangle \neq \emptyset$, then for every morphism f from a to b holds $(\overset{C}{\hookrightarrow})(f) = f$.

Let A be a category and let C be a non empty subcategory of A. Note that $\overset{C}{\hookrightarrow}$ is id-preserving and comp-preserving.

Let A be a category and let C be a non empty subcategory of A. Observe that $\overset{C}{\hookrightarrow}$ is precovariant.

Let A be a category and let C be a non empty subcategory of A. Then $\overset{C}{\hookrightarrow}$ is a strict covariant functor from C to A.

Let A, B be categories, let C be a non empty subcategory of A, and let F be a covariant functor from A to B. Then $F{\restriction}C$ is a strict covariant functor from C to B.

Let A, B be categories, let C be a non empty subcategory of A, and let F be a contravariant functor from A to B. Then $F{\restriction}C$ is a strict contravariant functor from C to B.

Next we state several propositions:

(29) Let A, B be categories, C be a non empty subcategory of A, F be a functor structure from A to B, a be an object of A, and c be an object of C. If $c = a$, then $(F{\restriction}C)(c) = F(a)$.

(30) Let A, B be categories, C be a non empty subcategory of A, F be a covariant functor from A to B, a, b be objects of A, and c, d be objects of C. Suppose $c = a$ and $d = b$ and $\langle c,d\rangle \neq \emptyset$. Let f be a morphism from a to b and g be a morphism from c to d. If $g = f$, then $(F{\restriction}C)(g) = F(f)$.

(31) Let A, B be categories, C be a non empty subcategory of A, F be a contravariant functor from A to B, a, b be objects of A, and c, d be objects of C. Suppose $c = a$ and $d = b$ and $\langle c, d \rangle \neq \emptyset$. Let f be a morphism from a to b and g be a morphism from c to d. If $g = f$, then $(F \restriction C)(g) = F(f)$.

(32) Let A, B be non empty graphs and F be a bimap structure from A into B. Suppose F is precovariant and one-to-one. Let a, b be objects of A. If $F(a) = F(b)$, then $a = b$.

(33) Let A, B be non empty reflexive graphs and F be a feasible precovariant functor structure from A to B. Suppose F is faithful. Let a, b be objects of A. Suppose $\langle a, b \rangle \neq \emptyset$. Let f, g be morphisms from a to b. If $F(f) = F(g)$, then $f = g$.

(34) Let A, B be non empty graphs and F be a precovariant functor structure from A to B. Suppose F is surjective. Let a, b be objects of B. Suppose $\langle a, b \rangle \neq \emptyset$. Let f be a morphism from a to b. Then there exist objects c, d of A and there exists a morphism g from c to d such that $a = F(c)$ and $b = F(d)$ and $\langle c, d \rangle \neq \emptyset$ and $f = F(g)$.

(35) Let A, B be non empty graphs and F be a bimap structure from A into B. Suppose F is precontravariant and one-to-one. Let a, b be objects of A. If $F(a) = F(b)$, then $a = b$.

(36) Let A, B be non empty reflexive graphs and F be a feasible precontravariant functor structure from A to B. Suppose F is faithful. Let a, b be objects of A. Suppose $\langle a, b \rangle \neq \emptyset$. Let f, g be morphisms from a to b. If $F(f) = F(g)$, then $f = g$.

(37) Let A, B be non empty graphs and F be a precontravariant functor structure from A to B. Suppose F is surjective. Let a, b be objects of B. Suppose $\langle a, b \rangle \neq \emptyset$. Let f be a morphism from a to b. Then there exist objects c, d of A and there exists a morphism g from c to d such that $b = F(c)$ and $a = F(d)$ and $\langle c, d \rangle \neq \emptyset$ and $f = F(g)$.

5. Isomorphisms under Arbitrary Functor

Let A, B be categories, let F be a functor structure from A to B, and let A', B' be categories. We say that A' and B' are isomorphic under F if and only if the conditions (Def. 4) are satisfied.

(Def. 4)(i) A' is a subcategory of A,

(ii) B' is a subcategory of B, and

(iii) there exists a covariant functor G from A' to B' such that G is bijective and for every object a' of A' and for every object a of A such that $a' = a$ holds $G(a') = F(a)$ and for all objects b', c' of A' and for all objects b, c of A such that $\langle b', c' \rangle \neq \emptyset$ and $b' = b$ and $c' = c$ and for every morphism f' from b' to c' and for every morphism f from b to c such that $f' = f$ holds $G(f') = (\text{Morph-Map}_F(b, c))(f)$.

We say that A' and B' are anti-isomorphic under F if and only if the conditions (Def. 5) are satisfied.

(Def. 5)(i) A' is a subcategory of A,

(ii) B' is a subcategory of B, and

(iii) there exists a contravariant functor G from A' to B' such that G is bijective and for every object a' of A' and for every object a of A such that $a' = a$ holds $G(a') = F(a)$ and for all objects b', c' of A' and for all objects b, c of A such that $\langle b', c' \rangle \neq \emptyset$ and $b' = b$ and $c' = c$ and for every morphism f' from b' to c' and for every morphism f from b to c such that $f' = f$ holds $G(f') = (\text{Morph-Map}_F(b, c))(f)$.

One can prove the following propositions:

(38) Let A, B, A_1, B_1 be categories and F be a functor structure from A to B. If A_1 and B_1 are isomorphic under F, then A_1 and B_1 are isomorphic.

(39) Let A, B, A_1, B_1 be categories and F be a functor structure from A to B. Suppose A_1 and B_1 are anti-isomorphic under F. Then A_1, B_1 are anti-isomorphic.

(40) Let A, B be categories and F be a covariant functor from A to B. If A and B are isomorphic under F, then F is bijective.

(41) Let A, B be categories and F be a contravariant functor from A to B. If A and B are anti-isomorphic under F, then F is bijective.

(42) Let A, B be categories and F be a covariant functor from A to B. If F is bijective, then A and B are isomorphic under F.

(43) Let A, B be categories and F be a contravariant functor from A to B. If F is bijective, then A and B are anti-isomorphic under F.

Now we present two schemes. The scheme *CoBijectRestriction* deals with non empty categories $\mathcal{A}$, $\mathcal{B}$, a covariant functor $\mathcal{C}$ from $\mathcal{A}$ to $\mathcal{B}$, a non empty subcategory $\mathcal{D}$ of $\mathcal{A}$, and a non empty subcategory $\mathcal{E}$ of $\mathcal{B}$, and states that:

$\mathcal{D}$ and $\mathcal{E}$ are isomorphic under $\mathcal{C}$

provided the parameters meet the following conditions:

- $\mathcal{C}$ is bijective,
- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{D}$ iff $\mathcal{C}(a)$ is an object of $\mathcal{E}$, and
- Let a, b be objects of $\mathcal{A}$. Suppose $\langle a,b\rangle \neq \emptyset$. Let a_1, b_1 be objects of $\mathcal{D}$. Suppose $a_1 = a$ and $b_1 = b$. Let a_2, b_2 be objects of $\mathcal{E}$. Suppose $a_2 = \mathcal{C}(a)$ and $b_2 = \mathcal{C}(b)$. Let f be a morphism from a to b. Then $f \in \langle a_1,b_1\rangle$ if and only if $\mathcal{C}(f) \in \langle a_2,b_2\rangle$.

The scheme *ContraBijectRestriction* deals with non empty categories $\mathcal{A}$, $\mathcal{B}$, a contravariant functor $\mathcal{C}$ from $\mathcal{A}$ to $\mathcal{B}$, a non empty subcategory $\mathcal{D}$ of $\mathcal{A}$, and a non empty subcategory $\mathcal{E}$ of $\mathcal{B}$, and states that:

$\mathcal{D}$ and $\mathcal{E}$ are anti-isomorphic under $\mathcal{C}$

provided the following conditions are satisfied:

- $\mathcal{C}$ is bijective,
- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{D}$ iff $\mathcal{C}(a)$ is an object of $\mathcal{E}$, and
- Let a, b be objects of $\mathcal{A}$. Suppose $\langle a,b\rangle \neq \emptyset$. Let a_1, b_1 be objects of $\mathcal{D}$. Suppose $a_1 = a$ and $b_1 = b$. Let a_2, b_2 be objects of $\mathcal{E}$. Suppose $a_2 = \mathcal{C}(a)$ and $b_2 = \mathcal{C}(b)$. Let f be a morphism from a to b. Then $f \in \langle a_1,b_1\rangle$ if and only if $\mathcal{C}(f) \in \langle b_2,a_2\rangle$.

We now state a number of propositions:

(44) For every category A and for every non empty subcategory B of A holds B and B are isomorphic under id_A.

(45) For all functions f, g such that $f \subseteq g$ holds $\curvearrowleft f \subseteq \curvearrowleft g$.

(46) For all functions f, g such that $\mathrm{dom}\, f$ is a binary relation and $\curvearrowleft f \subseteq \curvearrowleft g$ holds $f \subseteq g$.

(47) Let I, J be sets, A be a many sorted set indexed by $[:I, I:]$, and B be a many sorted set indexed by $[:J, J:]$. If $A \dot{\subseteq} B$, then $\curvearrowleft A \dot{\subseteq} \curvearrowleft B$.

(48) Let A be a transitive non empty category structure and B be a transitive non empty substructure of A. Then B^{op} is a substructure of A^{op}.

(49) For every category A and for every non empty subcategory B of A holds B^{op} is a subcategory of A^{op}.

(50) Let A be a category and B be a non empty subcategory of A. Then B and B^{op} are anti-isomorphic under the dualizing functor from A into A^{op}.

(51) Let A_1, A_2 be categories and F be a covariant functor from A_1 to A_2. Suppose F is bijective. Let B_1 be a non empty subcategory of A_1 and B_2 be a non empty subcategory of A_2. Suppose B_1 and B_2 are isomorphic under F. Then B_2 and B_1 are isomorphic under F^{-1}.

(52) Let A_1, A_2 be categories and F be a contravariant functor from A_1 to A_2. Suppose F is bijective. Let B_1 be a non empty subcategory of A_1 and B_2 be a non empty subcategory of A_2. Suppose B_1 and B_2 are anti-isomorphic under F. Then B_2 and B_1 are anti-isomorphic under F^{-1}.

(53) Let A_1, A_2, A_3 be categories, F be a covariant functor from A_1 to A_2, G be a covariant functor from A_2 to A_3, B_1 be a non empty subcategory of A_1, B_2 be a non empty subcategory of A_2, and B_3 be a non empty subcategory of A_3. Suppose B_1 and B_2 are isomorphic under F and B_2 and B_3 are isomorphic under G. Then B_1 and B_3 are isomorphic under $G \cdot F$.

(54) Let A_1, A_2, A_3 be categories, F be a contravariant functor from A_1 to A_2, G be a covariant functor from A_2 to A_3, B_1 be a non empty subcategory of A_1, B_2 be a non empty subcategory of A_2, and B_3 be a non empty subcategory of A_3. Suppose B_1 and B_2 are anti-isomorphic under F and B_2 and B_3 are isomorphic under G. Then B_1 and B_3 are anti-isomorphic under $G \cdot F$.

(55) Let A_1, A_2, A_3 be categories, F be a covariant functor from A_1 to A_2, G be a contravariant functor from A_2 to A_3, B_1 be a non empty subcategory of A_1, B_2 be a non empty subcategory of A_2, and B_3 be a non empty subcategory of A_3. Suppose B_1 and B_2 are isomorphic under F and B_2 and B_3 are anti-isomorphic under G. Then B_1 and B_3 are anti-isomorphic under $G \cdot F$.

(56) Let A_1, A_2, A_3 be categories, F be a contravariant functor from A_1 to A_2, G be a contravariant functor from A_2 to A_3, B_1 be a non empty subcategory of A_1, B_2 be a non empty subcategory of A_2, and B_3 be a non empty subcategory of A_3. Suppose B_1 and B_2 are anti-isomorphic under F and B_2 and B_3 are anti-isomorphic under G. Then B_1 and B_3 are isomorphic under $G \cdot F$.

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