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Introduction to Arithmetic of Real Numbers

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The articles [4], [3], [7], [1], [2], [5], and [6] provide the notation and terminology for this paper.

1. Main Definitions

Let r be a number. We say that r is real if and only if:

(Def. 1) $r \in \mathbb{R}$.

Let us mention that every number which is natural is also real and every number which is real is also complex.

Let us note that there exists a number which is real.

Let x, y be real numbers. The predicate $x \leq y$ is defined by:

(Def. 2)(i) There exist elements x', y' of $\mathbb{R}_+$ such that $x = x'$ and $y = y'$ and $x' \leq y'$ if $x \in \mathbb{R}_+$ and $y \in \mathbb{R}_+$,

(ii) there exist elements x', y' of $\mathbb{R}_+$ such that $x = \langle 0, x' \rangle$ and $y = \langle 0, y' \rangle$ and $y' \leq x'$ if $x \in [: \{0\}, \mathbb{R}_+ :]$ and $y \in [: \{0\}, \mathbb{R}_+ :]$,

(iii) $y \in \mathbb{R}_+$ and $x \in [: \{0\}, \mathbb{R}_+ :]$, otherwise.

Let us notice that the predicate $x \leq y$ is reflexive and connected. We introduce $y \geq x$ as a synonym of $x \leq y$. We introduce $y < x$ and $x > y$ as antonyms of $x \leq y$.

Let x be a real number. We say that x is positive if and only if:

(Def. 3) $x > 0$.

We say that x is negative if and only if:

(Def. 4) $x < 0$.

Let x be a real number. One can verify that $-x$ is real and x^{-1} is real. Let y be a real number. Observe that $x + y$ is real and $x \cdot y$ is real.

Let x, y be real numbers. Note that $x - y$ is real and $\frac{x}{y}$ is real.

2. Clusters

One can check the following observations:

* every real number which is positive is also non negative and non zero,
* every real number which is non negative and non zero is also positive,

* every real number which is negative is also non positive and non zero,
* every real number which is non positive and non zero is also negative,
* every real number which is zero is also non negative and non positive, and
* every real number which is non negative and non positive is also zero.

One can check the following observations:

* there exists a real number which is positive,
* there exists a real number which is negative, and
* there exists a real number which is zero.

Let r, s be non negative real numbers. Note that $r+s$ is non negative.

Let r, s be non positive real numbers. Note that $r+s$ is non positive.

Let r be a positive real number and let s be a non negative real number. One can verify that $r+s$ is positive and $s+r$ is positive.

Let r be a negative real number and let s be a non positive real number. Note that $r+s$ is negative and $s+r$ is negative.

Let r be a non positive real number. Observe that $-r$ is non negative.

Let r be a non negative real number. One can verify that $-r$ is non positive.

Let r be a non negative real number and let s be a non positive real number. One can check that $r-s$ is non negative and $s-r$ is non positive.

Let r be a positive real number and let s be a non positive real number. Note that $r-s$ is positive and $s-r$ is negative.

Let r be a negative real number and let s be a non negative real number. Note that $r-s$ is negative and $s-r$ is positive.

Let r be a non positive real number and let s be a non negative real number. Observe that $r \cdot s$ is non positive and $s \cdot r$ is non positive.

Let r, s be non positive real numbers. One can check that $r \cdot s$ is non negative.

Let r, s be non negative real numbers. Note that $r \cdot s$ is non negative.

Let r be a non positive real number. Note that r^{-1} is non positive.

Let r be a non negative real number. One can check that r^{-1} is non negative.

Let r be a non negative real number and let s be a non positive real number. Note that $\frac{r}{s}$ is non positive and $\frac{s}{r}$ is non positive.

Let r, s be non negative real numbers. One can verify that $\frac{r}{s}$ is non negative.

Let r, s be non positive real numbers. Note that $\frac{r}{s}$ is non negative.

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