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Boolean Properties of Sets — Theorems

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The article [1] provides the notation and terminology for this paper.

1. MAIN PART

In this paper A, B, X, X', Y, Y', Z, V denote sets.

One can prove the following propositions:

(1) If $X \subseteq Y$ and $Y \subseteq Z$, then $X \subseteq Z$.

(2) $\emptyset \subseteq X$.

(3) If $X \subseteq \emptyset$, then $X = \emptyset$.

(4) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$.

(5) $(X \cup Y) \cup Z = X \cup Z \cup (Y \cup Z)$.

(6) $X \cup (X \cup Y) = X \cup Y$.

(7) $X \subseteq X \cup Y$.

(8) If $X \subseteq Z$ and $Y \subseteq Z$, then $X \cup Y \subseteq Z$.

(9) If $X \subseteq Y$, then $X \cup Z \subseteq Y \cup Z$.

(10) If $X \subseteq Y$, then $X \subseteq Z \cup Y$.

(11) If $X \cup Y \subseteq Z$, then $X \subseteq Z$.

(12) If $X \subseteq Y$, then $X \cup Y = Y$.

(13) If $X \subseteq Y$ and $Z \subseteq V$, then $X \cup Z \subseteq Y \cup V$.

(14) If $Y \subseteq X$ and $Z \subseteq X$ and for every V such that $Y \subseteq V$ and $Z \subseteq V$ holds $X \subseteq V$, then $X = Y \cup Z$.

(15) If $X \cup Y = \emptyset$, then $X = \emptyset$.

(16) $(X \cap Y) \cap Z = X \cap (Y \cap Z)$.

(17) $X \cap Y \subseteq X$.

(18) If $X \subseteq Y \cap Z$, then $X \subseteq Y$.

(19) If $Z \subseteq X$ and $Z \subseteq Y$, then $Z \subseteq X \cap Y$.

(20) If $X \subseteq Y$ and $X \subseteq Z$ and for every V such that $V \subseteq Y$ and $V \subseteq Z$ holds $V \subseteq X$, then $X = Y \cap Z$.

(21) $X \cap (X \cup Y) = X$.

(22) $X \cup X \cap Y = X$.

(23) $X \cap (Y \cup Z) = X \cap Y \cup X \cap Z$.

(24) $X \cup Y \cap Z = (X \cup Y) \cap (X \cup Z)$.

(25) $X \cap Y \cup Y \cap Z \cup Z \cap X = (X \cup Y) \cap (Y \cup Z) \cap (Z \cup X)$.

(26) If $X \subseteq Y$, then $X \cap Z \subseteq Y \cap Z$.

(27) If $X \subseteq Y$ and $Z \subseteq V$, then $X \cap Z \subseteq Y \cap V$.

(28) If $X \subseteq Y$, then $X \cap Y = X$.

(29) $X \cap Y \subseteq X \cup Z$.

(30) If $X \subseteq Z$, then $X \cup Y \cap Z = (X \cup Y) \cap Z$.

(31) $X \cap Y \cup X \cap Z \subseteq Y \cup Z$.

(32) If $X \setminus Y = Y \setminus X$, then $X = Y$.

(33) If $X \subseteq Y$, then $X \setminus Z \subseteq Y \setminus Z$.

(34) If $X \subseteq Y$, then $Z \setminus Y \subseteq Z \setminus X$.

(35) If $X \subseteq Y$ and $Z \subseteq V$, then $X \setminus V \subseteq Y \setminus Z$.

(36) $X \setminus Y \subseteq X$.

(37) $X \setminus Y = \emptyset$ iff $X \subseteq Y$.

(38) If $X \subseteq Y \setminus X$, then $X = \emptyset$.

(39) $X \cup (Y \setminus X) = X \cup Y$.

(40) $(X \cup Y) \setminus Y = X \setminus Y$.

(41) $X \setminus Y \setminus Z = X \setminus (Y \cup Z)$.

(42) $(X \cup Y) \setminus Z = (X \setminus Z) \cup (Y \setminus Z)$.

(43) If $X \subseteq Y \cup Z$, then $X \setminus Y \subseteq Z$.

(44) If $X \setminus Y \subseteq Z$, then $X \subseteq Y \cup Z$.

(45) If $X \subseteq Y$, then $Y = X \cup (Y \setminus X)$.

(46) $X \setminus (X \cup Y) = \emptyset$.

(47) $X \setminus X \cap Y = X \setminus Y$.

(48) $X \setminus (X \setminus Y) = X \cap Y$.

(49) $X \cap (Y \setminus Z) = X \cap Y \setminus Z$.

(50) $X \cap (Y \setminus Z) = X \cap Y \setminus X \cap Z$.

(51) $X \cap Y \cup (X \setminus Y) = X$.

(52) $X \setminus (Y \setminus Z) = (X \setminus Y) \cup X \cap Z$.

(53) $X \setminus (Y \cup Z) = (X \setminus Y) \cap (X \setminus Z)$.

(54) $X \setminus Y \cap Z = (X \setminus Y) \cup (X \setminus Z)$.

(55) $(X \cup Y) \setminus X \cap Y = (X \setminus Y) \cup (Y \setminus X)$.

(56) If $X \subset Y$ and $Y \subset Z$, then $X \subset Z$.

(57) $X \not\subset Y$ or $Y \not\subset X$.

(58) If $X \subset Y$ and $Y \subseteq Z$, then $X \subset Z$.

(59) If $X \subseteq Y$ and $Y \subset Z$, then $X \subset Z$.

(60) If $X \subseteq Y$, then $Y \not\subset X$.

(61) If $X \neq \emptyset$, then $\emptyset \subset X$.

(62) $X \not\subset \emptyset$.

(63) If $X \subseteq Y$ and Y misses Z, then X misses Z.

(64) If $A \subseteq X$ and $B \subseteq Y$ and X misses Y, then A misses B.

(65) X misses $\emptyset$.

(66) X meets X iff $X \neq \emptyset$.

(67) If $X \subseteq Y$ and $X \subseteq Z$ and Y misses Z, then $X = \emptyset$.

(68) For every non empty set A such that $A \subseteq Y$ and $A \subseteq Z$ holds Y meets Z.

(69) For every non empty set A such that $A \subseteq Y$ holds A meets Y.

(70) X meets $Y \cup Z$ iff X meets Y or X meets Z.

(71) If $X \cup Y = Z \cup Y$ and X misses Y and Z misses Y, then $X = Z$.

(72) If $X' \cup Y' = X \cup Y$ and X misses X' and Y misses Y', then $X = Y'$.

(73) If $X \subseteq Y \cup Z$ and X misses Z, then $X \subseteq Y$.

(74) If X meets $Y \cap Z$, then X meets Y.

(75) If X meets Y, then $X \cap Y$ meets Y.

(76) If Y misses Z, then $X \cap Y$ misses $X \cap Z$.

(77) If X meets Y and $X \subseteq Z$, then X meets $Y \cap Z$.

(78) If X misses Y, then $X \cap (Y \cup Z) = X \cap Z$.

(79) $X \setminus Y$ misses Y.

(80) If X misses Y, then X misses $Y \setminus Z$.

(81) If X misses $Y \setminus Z$, then Y misses $X \setminus Z$.

(82) $X \setminus Y$ misses $Y \setminus X$.

(83) X misses Y iff $X \setminus Y = X$.

(84) If X meets Y and X misses Z, then X meets $Y \setminus Z$.

(85) If $X \subseteq Y$, then X misses $Z \setminus Y$.

(86) If $X \subseteq Y$ and X misses Z, then $X \subseteq Y \setminus Z$.

(87) If Y misses Z, then $(X \setminus Y) \cup Z = (X \cup Z) \setminus Y$.

(88) If X misses Y, then $(X \cup Y) \setminus Y = X$.

(89) $X \cap Y$ misses $X \setminus Y$.

(90) $X \setminus X \cap Y$ misses Y.

(91) $(X \dot{-} Y) \dot{-} Z = X \dot{-} (Y \dot{-} Z)$.

(92) $X \dot{-} X = \emptyset$.

(93) $X \cup Y = (X \dot{-} Y) \cup X \cap Y$.

(94) $X \cup Y = X \dot{-} Y \dot{-} X \cap Y$.

(95) $X \cap Y = X \dot{-} Y \dot{-} (X \cup Y)$.

(96) $X \setminus Y \subseteq X \dot{-} Y$.

(97) If $X \setminus Y \subseteq Z$ and $Y \setminus X \subseteq Z$, then $X \dot{-} Y \subseteq Z$.

(98) $X \cup Y = X \dot{-} (Y \setminus X)$.

(99) $(X \dot{-} Y) \setminus Z = (X \setminus (Y \cup Z)) \cup (Y \setminus (X \cup Z))$.

(100) $X \setminus Y = X \dot{-} X \cap Y$.

(101) $X \dot{-} Y = (X \cup Y) \setminus X \cap Y$.

(102) $X \setminus (Y \dot{-} Z) = (X \setminus (Y \cup Z)) \cup X \cap Y \cap Z$.

(103) $X \cap Y$ misses $X \dot{-} Y$.

(104) $X \subset Y$ or $X = Y$ or $Y \subset X$ iff X and Y are $\subseteq$-comparable.

2. Appendix

Next we state several propositions:

(105) For all sets X, Y such that $X \subset Y$ holds $Y \setminus X \neq \emptyset$.

(106) If $X \subseteq A \setminus B$, then $X \subseteq A$ and X misses B.

(107) $X \subseteq A \dot{-} B$ iff $X \subseteq A \cup B$ and X misses $A \cap B$.

(108) If $X \subseteq A$, then $X \cap Y \subseteq A$.

(109) If $X \subseteq A$, then $X \setminus Y \subseteq A$.

(110) If $X \subseteq A$ and $Y \subseteq A$, then $X \dot{-} Y \subseteq A$.

(111) $X \cap Z \setminus Y \cap Z = (X \setminus Y) \cap Z$.

(112) $X \cap Z \dot{-} Y \cap Z = (X \dot{-} Y) \cap Z$.

3. Appendix 2

One can prove the following proposition

(113) $(X \cup Y \cup Z) \cup V = X \cup (Y \cup Z \cup V)$.

REFERENCES

[1] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

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