JOURNAL OF FORMALIZED MATHEMATICS
Volume 1, Released 1989, Published 2003
Inst. of Computer Science, Univ. of Białystok

Zermelo's Theorem[1]

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Summary. The article contains direct proof of Zermelo's theorem about the existence of a well ordering for any set and the lemma the proof depends on.

MML Identifier: WELLSET1.

WWW: http://mizar.org/JFM/Vol1/wellset1.html

The articles [4], [3], [5], [2], and [1] provide the notation and terminology for this paper.

We adopt the following rules: x, y, B, D, N, X, Y denote sets, R, W denote binary relations, and F denotes a function.

We now state two propositions:

(1) $x \in \operatorname{field} R$ iff there exists y such that $\langle x, y \rangle \in R$ or $\langle y, x \rangle \in R$.

(3)[1] If $X \neq \emptyset$ and $Y \neq \emptyset$ and $W = [: X, Y :]$, then $\operatorname{field} W = X \cup Y$.

The scheme *R Separation* deals with a set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

There exists B such that for every binary relation R holds $R \in B$ iff $R \in \mathcal{A}$ and $\mathcal{P}[R]$

for all values of the parameters.

One can prove the following four propositions:

(6)[2] For all x, y, W such that $x \in \operatorname{field} W$ and $y \in \operatorname{field} W$ and W is well-ordering holds if $x \notin W\text{-Seg}(y)$, then $\langle y, x \rangle \in W$.

(7) For all x, y, W such that $x \in \operatorname{field} W$ and $y \in \operatorname{field} W$ and W is well-ordering holds if $x \in W\text{-Seg}(y)$, then $\langle y, x \rangle \notin W$.

(8) Let given F, D. Suppose that for every X such that $X \in D$ holds $F(X) \notin X$ and $F(X) \in \bigcup D$. Then there exists R such that $\operatorname{field} R \subseteq \bigcup D$ and R is well-ordering and $\operatorname{field} R \notin D$ and for every y such that $y \in \operatorname{field} R$ holds $R\text{-Seg}(y) \in D$ and $F(R\text{-Seg}(y)) = y$.

(9) For every N there exists R such that R is well-ordering and $\operatorname{field} R = N$.

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[3] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[1]Supported by RPBP.III-24.C9.

[1] The proposition (2) has been removed.

[2] The propositions (4) and (5) have been removed.

[4] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[5] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

Received October 27, 1989

Published January 2, 2004