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Lim-Inf Convergence[1]

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Summary. This work continues the formalization of [8]. Theorems from Chapter III, Section 3, pp. 158–159 are proved.

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The articles [13], [7], [17], [18], [19], [5], [6], [11], [12], [16], [1], [2], [3], [4], [14], [10], [15], and [9] provide the notation and terminology for this paper.

One can prove the following propositions:

(1) For every complete lattice L and for every net N in L holds $\inf N \leq \liminf N$.

(2) Let L be a complete lattice, N be a net in L, and x be an element of L. Suppose that for every subnet M of N holds $x = \liminf M$. Then $x = \liminf N$ and for every subnet M of N holds $x \geq \inf M$.

(3) Let L be a complete lattice, N be a net in L, and x be an element of L. Suppose $N \in \mathrm{NetUniv}(L)$. Suppose that for every subnet M of N such that $M \in \mathrm{NetUniv}(L)$ holds $x = \liminf M$. Then $x = \liminf N$ and for every subnet M of N such that $M \in \mathrm{NetUniv}(L)$ holds $x \geq \inf M$.

Let N be a non empty relational structure and let f be a map from N into N. We say that f is greater or equal to id if and only if:

(Def. 1) For every element j of N holds $j \leq f(j)$.

The following three propositions are true:

(4) For every reflexive non empty relational structure N holds id_N is greater or equal to id.

(5) Let N be a directed non empty relational structure and x, y be elements of N. Then there exists an element z of N such that $x \leq z$ and $y \leq z$.

(6) For every directed non empty relational structure N holds there exists a map from N into N which is greater or equal to id.

Let N be a directed non empty relational structure. One can verify that there exists a map from N into N which is greater or equal to id.

Let N be a reflexive non empty relational structure. Observe that there exists a map from N into N which is greater or equal to id.

Let L be a non empty 1-sorted structure, let N be a non empty net structure over L, and let f be a map from N into N. The functor $N \cdot f$ yields a strict non empty net structure over L and is defined by the conditions (Def. 2).

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(Def. 2)(i) The relational structure of $N \cdot f =$ the relational structure of N, and

(ii) the mapping of $N \cdot f =$ (the mapping of $N) \cdot f$.

We now state three propositions:

(7) Let L be a non empty 1-sorted structure, N be a non empty net structure over L, and f be a map from N into N. Then the carrier of $N \cdot f =$ the carrier of N.

(8) Let L be a non empty 1-sorted structure, N be a non empty net structure over L, and f be a map from N into N. Then $N \cdot f = \langle$the carrier of N, the internal relation of N, (the mapping of $N) \cdot f\rangle$.

(9) Let L be a non empty 1-sorted structure, N be a transitive directed non empty relational structure, and f be a function from the carrier of N into the carrier of L. Then $\langle$the carrier of N, the internal relation of N, $f\rangle$ is a net in L.

Let L be a non empty 1-sorted structure, let N be a transitive directed non empty relational structure, and let f be a function from the carrier of N into the carrier of L. Observe that $\langle$the carrier of N, the internal relation of N, $f\rangle$ is transitive, directed, and non empty.

One can prove the following proposition

(10) Let L be a non empty 1-sorted structure, N be a net in L, and p be a map from N into N. Then $N \cdot p$ is a net in L.

Let L be a non empty 1-sorted structure, let N be a net in L, and let p be a map from N into N. Observe that $N \cdot p$ is transitive and directed.

The following two propositions are true:

(11) Let L be a non empty 1-sorted structure, N be a net in L, and p be a map from N into N. If $N \in \mathrm{NetUniv}(L)$, then $N \cdot p \in \mathrm{NetUniv}(L)$.

(12) Let L be a non empty 1-sorted structure and N, M be nets in L. Suppose the net structure of $N =$ the net structure of M. Then M is a subnet of N.

Let L be a non empty 1-sorted structure and let N be a net in L. Note that there exists a subnet of N which is strict.

The following proposition is true

(13) Let L be a non empty 1-sorted structure, N be a net in L, and p be a greater or equal to id map from N into N. Then $N \cdot p$ is a subnet of N.

Let L be a non empty 1-sorted structure, let N be a net in L, and let p be a greater or equal to id map from N into N. Then $N \cdot p$ is a strict subnet of N.

We now state two propositions:

(14) Let L be a complete lattice, N be a net in L, and x be an element of L. Suppose $N \in \mathrm{NetUniv}(L)$. Suppose $x = \liminf N$ and for every subnet M of N such that $M \in \mathrm{NetUniv}(L)$ holds $x \geq \inf M$. Then $x = \liminf N$ and for every greater or equal to id map p from N into N holds $x \geq \inf(N \cdot p)$.

(15) Let L be a complete lattice, N be a net in L, and x be an element of L. Suppose $x = \liminf N$ and for every greater or equal to id map p from N into N holds $x \geq \inf(N \cdot p)$. Let M be a subnet of N. Then $x = \liminf M$.

Let L be a non empty relational structure. The lim inf convergence of L is a convergence class of L and is defined by the condition (Def. 3).

(Def. 3) Let N be a net in L. Suppose $N \in \mathrm{NetUniv}(L)$. Let x be an element of L. Then $\langle N, x\rangle \in$ the lim inf convergence of L if and only if for every subnet M of N holds $x = \liminf M$.

One can prove the following two propositions:

(16) Let L be a complete lattice, N be a net in L, and x be an element of L. Suppose $N \in \mathrm{NetUniv}(L)$. Then $\langle N, x \rangle \in$ the lim inf convergence of L if and only if for every subnet M of N such that $M \in \mathrm{NetUniv}(L)$ holds $x = \liminf M$.

(17) Let L be a non empty relational structure, N be a constant net in L, and M be a subnet of N. Then M is constant and the value of $N =$ the value of M.

Let L be a non empty relational structure. The functor $\xi(L)$ yielding a family of subsets of L is defined as follows:

(Def. 4) $\xi(L) =$ the topology of ConvergenceSpace(the lim inf convergence of L).

One can prove the following propositions:

(18) For every complete lattice L holds the lim inf convergence of L has (CONSTANTS) property.

(19) For every non empty relational structure L holds the lim inf convergence of L has (SUBNETS) property.

(20) For every continuous complete lattice L holds the lim inf convergence of L has (DIVERGENCE) property.

(21) Let L be a non empty relational structure and N, x be sets. If $\langle N, x \rangle \in$ the lim inf convergence of L, then $N \in \mathrm{NetUniv}(L)$.

(22) Let L be a non empty 1-sorted structure and C_1, C_2 be convergence classes of L. If $C_1 \subseteq C_2$, then the topology of ConvergenceSpace$(C_2) \subseteq$ the topology of ConvergenceSpace(C_1).

(23) Let L be a non empty reflexive relational structure. Then the lim inf convergence of $L \subseteq$ the Scott convergence of L.

(24) For all sets X, Y such that $X \subseteq Y$ holds $X \in$ the universe of Y.

(25) Let L be a non empty transitive reflexive relational structure and D be a directed non empty subset of L. Then $\mathrm{NetStr}(D) \in \mathrm{NetUniv}(L)$.

(26) For every complete lattice L and for every directed non empty subset D of L and for every subnet M of $\mathrm{NetStr}(D)$ holds $\liminf M = \sup D$.

(27) Let L be a non empty complete lattice and D be a directed non empty subset of L. Then $\langle \mathrm{NetStr}(D), \sup D \rangle \in$ the lim inf convergence of L.

(28) For every complete lattice L and for every subset U_1 of L such that $U_1 \in \xi(L)$ holds U_1 is property(S).

(29) For every non empty reflexive relational structure L and for every subset A of L such that $A \in \sigma(L)$ holds $A \in \xi(L)$.

(30) For every complete lattice L and for every subset A of L such that A is upper holds if $A \in \xi(L)$, then $A \in \sigma(L)$.

(31) Let L be a complete lattice and A be a subset of L. Suppose A is lower. Then $A^{\mathrm{c}} \in \xi(L)$ if and only if A is closed under directed sups.

REFERENCES

[1] Grzegorz Bancerek. Complete lattices. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/lattice3.html.

[2] Grzegorz Bancerek. Bounds in posets and relational substructures. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/yellow_0.html.

[3] Grzegorz Bancerek. Directed sets, nets, ideals, filters, and maps. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/waybel_0.html.

[4] Grzegorz Bancerek. The "way-below" relation. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/waybel_3.html.

[5] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[6] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[7] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[8] G. Gierz, K.H. Hofmann, K. Keimel, J.D. Lawson, M. Mislove, and D.S. Scott. *A Compendium of Continuous Lattices*. Springer-Verlag, Berlin, Heidelberg, New York, 1980.

[9] Adam Grabowski. Scott-continuous functions. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/waybel17.html.

[10] Artur Korniłowicz. On the topological properties of meet-continuous lattices. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/waybel_9.html.

[11] Michał Muzalewski. Categories of groups. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/grcat_1.html.

[12] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/pre_topc.html.

[13] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[14] Andrzej Trybulec. Moore-Smith convergence. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/yellow_6.html.

[15] Andrzej Trybulec. Scott topology. *Journal of Formalized Mathematics*, 9, 1997. http://mizar.org/JFM/Vol9/waybel11.html.

[16] Wojciech A. Trybulec. Partially ordered sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/orders_1.html.

[17] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[18] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

[19] Edmund Woronowicz. Relations defined on sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relset_1.html.

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