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Basic Notation of Universal Algebra

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Summary. We present the basic notation of universal algebra.

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The articles [5], [7], [6], [8], [2], [1], [4], and [3] provide the notation and terminology for this paper.

For simplicity, we follow the rules: A denotes a set, x, y denote finite sequences of elements of A, h denotes a partial function from A^* to A, and n denotes a natural number.

Let us consider A and let I_1 be a partial function from A^* to A. We say that I_1 is homogeneous if and only if:

(Def. 1) For all x, y such that $x \in \mathrm{dom}\, I_1$ and $y \in \mathrm{dom}\, I_1$ holds $\mathrm{len}\, x = \mathrm{len}\, y$.

Let us consider A and let I_1 be a partial function from A^* to A. We say that I_1 is quasi total if and only if:

(Def. 2) For all x, y such that $\mathrm{len}\, x = \mathrm{len}\, y$ and $x \in \mathrm{dom}\, I_1$ holds $y \in \mathrm{dom}\, I_1$.

Let A be a non empty set. Observe that there exists a partial function from A^* to A which is homogeneous, quasi total, and non empty.

We now state three propositions:

(1) h is non empty iff $\mathrm{dom}\, h \neq \emptyset$.

(2) Let A be a non empty set and a be an element of A. Then $\{\varepsilon_A\} \longmapsto a$ is a homogeneous quasi total non empty partial function from A^* to A.

(3) For every non empty set A and for every element a of A holds $\{\varepsilon_A\} \longmapsto a$ is an element of $A^* \dot{\rightarrow} A$.

Let us consider A. A finite sequence of operational functions of A is a finite sequence of elements of $A^* \dot{\rightarrow} A$.

We introduce universal algebra structures which are extensions of 1-sorted structure and are systems

⟨ a carrier, a characteristic ⟩,

where the carrier is a set and the characteristic is a finite sequence of operational functions of the carrier.

Let us mention that there exists a universal algebra structure which is non empty and strict.

Let D be a non empty set and let c be a finite sequence of operational functions of D. Note that $\langle D, c \rangle$ is non empty.

Let us consider A and let I_1 be a finite sequence of operational functions of A. We say that I_1 is homogeneous if and only if:

(Def. 4)[1] For all n, h such that $n \in \operatorname{dom} I_1$ and $h = I_1(n)$ holds h is homogeneous.

Let us consider A and let I_1 be a finite sequence of operational functions of A. We say that I_1 is quasi total if and only if:

(Def. 5) For all n, h such that $n \in \operatorname{dom} I_1$ and $h = I_1(n)$ holds h is quasi total.

Let F be a function. Let us observe that F is non-empty if and only if:

(Def. 6) For every set n such that $n \in \operatorname{dom} F$ holds $F(n)$ is non empty.

Let A be a non empty set and let x be an element of $A^* \dot{\rightarrow} A$. Then $\langle x \rangle$ is a finite sequence of operational functions of A.

Let A be a non empty set. One can verify that there exists a finite sequence of operational functions of A which is homogeneous, quasi total, and non-empty.

Let I_1 be a universal algebra structure. We say that I_1 is partial if and only if:

(Def. 7) The characteristic of I_1 is homogeneous.

We say that I_1 is quasi total if and only if:

(Def. 8) The characteristic of I_1 is quasi total.

We say that I_1 is non-empty if and only if:

(Def. 9) The characteristic of $I_1 \neq \emptyset$ and the characteristic of I_1 is non-empty.

In the sequel A denotes a non empty set and x denotes a finite sequence of elements of A.

Next we state the proposition

(4) For every element x of $A^* \dot{\rightarrow} A$ such that $x = \{\varepsilon_A\} \longmapsto a$ holds $\langle x \rangle$ is homogeneous, quasi total, and non-empty.

Let us note that there exists a universal algebra structure which is quasi total, partial, non-empty, strict, and non empty.

Let U_1 be a partial universal algebra structure. Observe that the characteristic of U_1 is homogeneous.

Let U_1 be a quasi total universal algebra structure. Note that the characteristic of U_1 is quasi total.

Let U_1 be a non-empty universal algebra structure. Note that the characteristic of U_1 is non-empty and non empty.

A universal algebra is a quasi total partial non-empty non empty universal algebra structure.

In the sequel U_1 denotes a partial non-empty non empty universal algebra structure.

Let us consider A and let f be a homogeneous non empty partial function from A^* to A. The functor arity f yielding a natural number is defined as follows:

(Def. 10) If $x \in \operatorname{dom} f$, then $\operatorname{arity} f = \operatorname{len} x$.

Next we state the proposition

(5) Let given U_1 and given n. Suppose $n \in \operatorname{dom}$(the characteristic of U_1). Then (the characteristic of $U_1)(n)$ is a partial function from (the carrier of $U_1)^*$ to the carrier of U_1.

Let us consider U_1. The functor signature U_1 yielding a finite sequence of elements of $\mathbb{N}$ is defined by the conditions (Def. 11).

(Def. 11)(i) $\operatorname{len} \operatorname{signature} U_1 = \operatorname{len}$(the characteristic of U_1), and

(ii) for every n such that $n \in \operatorname{dom} \operatorname{signature} U_1$ and for every homogeneous non empty partial function h from (the carrier of $U_1)^*$ to the carrier of U_1 such that $h =$ (the characteristic of $U_1)(n)$ holds $(\operatorname{signature} U_1)(n) = \operatorname{arity} h$.

[1] The definition (Def. 3) has been removed.

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