JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

Translations in Affine Planes [1]

Henryk Oryszczyszyn
Warsaw University
Białystok

Krzysztof Prażmowski
Warsaw University
Białystok

Summary. Connections between Minor Desargues Axiom and the transitivity of translation groups are investigated. A formal proof of the theorem which establishes the equivalence of these two properties of affine planes is given. We also prove that, under additional requirement, the plane in question satisfies Fano Axiom; its translation group is uniquely two-divisible.

MML Identifier: `TRANSLAC`.

WWW: `http://mizar.org/JFM/Vol2/translac.html`

The articles [2], [1], [3], [5], [6], [4], and [7] provide the notation and terminology for this paper.

We follow the rules: A_1 is an affine space and a, b, c, d, p, q, r are elements of A_1.

Let us consider A_1. We say that A_1 is Fanoian if and only if:

(Def. 1) For all a, b, c, d such that $a,b \upuparrows c,d$ and $a,c \upuparrows b,d$ and $a,d \upuparrows b,c$ holds $\mathbf{L}(a,b,c)$.

We introduce A_1 satisfies Fano Axiom as a synonym of A_1 is Fanoian.

We now state the proposition

(2)[1] Given a, b, c such that $\mathbf{L}(a,b,c)$ and $a \neq b$ and $a \neq c$ and $b \neq c$. Let given p, q. If $p \neq q$, then there exists r such that $\mathbf{L}(p,q,r)$ and $p \neq r$ and $q \neq r$.

We adopt the following rules: A_2 is an affine plane, a, a', b, b', c, c', d, p, q, r, x, y are elements of A_2, and f, g, f_1, f_2 are permutations of the carrier of A_2.

We now state a number of propositions:

(4)[2] If A_2 satisfies Fano Axiom and $a,b \upuparrows c,d$ and $a,c \upuparrows b,d$ and not $\mathbf{L}(a,b,c)$, then there exists p such that $\mathbf{L}(b,c,p)$ and $\mathbf{L}(a,d,p)$.

(5) If f is a translation and not $\mathbf{L}(a,f(a),x)$ and $a,f(a) \upuparrows x,y$ and $a,x \upuparrows f(a),y$, then $y = f(x)$.

(6) A_2 satisfies **des** if and only if for all a, a', b, c, b', c' such that not $\mathbf{L}(a,a',b)$ and not $\mathbf{L}(a,a',c)$ and $a,a' \upuparrows b,b'$ and $a,a' \upuparrows c,c'$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ holds $b,c \upuparrows b',c'$.

(7) There exists f such that f is a translation and $f(a) = a$.

(8) If for all p, q, r such that $p \neq q$ and $\mathbf{L}(p,q,r)$ holds $r = p$ or $r = q$ and $a,b \upuparrows p,q$ and $a,p \upuparrows b,q$ and not $\mathbf{L}(a,b,p)$, then $a,q \upuparrows b,p$.

(9) If A_2 satisfies **des**, then there exists f such that f is a translation and $f(a) = b$.

[1]Supported by RPBP.III-24.C2.
[1] The proposition (1) has been removed.
[2] The proposition (3) has been removed.

(10) If for all a, b there exists f such that f is a translation and $f(a) = b$, then A_2 satisfies **des**.

(11) If f is a translation and g is a translation and not $\mathbf{L}(a, f(a), g(a))$, then $f \cdot g = g \cdot f$.

(12) If A_2 satisfies **des** and f is a translation and g is a translation, then $f \cdot g = g \cdot f$.

(13) If f is a translation and g is a translation and $p, f(p) \upharpoonleft\upharpoonright p, g(p)$, then $p, f(p) \upharpoonleft\upharpoonright p, (f \cdot g)(p)$.

(14) Suppose A_2 satisfies Fano Axiom and A_2 satisfies **des** and f is a translation. Then there exists g such that g is a translation and $g \cdot g = f$.

(15) If A_2 satisfies Fano Axiom and f is a translation and $f \cdot f = \mathrm{id}_{\text{the carrier of } A_2}$, then $f = \mathrm{id}_{\text{the carrier of } A_2}$.

(16) Suppose that A_2 satisfies **des** and A_2 satisfies Fano Axiom and g is a translation and f_1 is a translation and f_2 is a translation and $g = f_1 \cdot f_1$ and $g = f_2 \cdot f_2$. Then $f_1 = f_2$.

REFERENCES

[1] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[2] Czesław Byliński. Partial functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/partfun1.html.

[3] Henryk Oryszczyszyn and Krzysztof Prażmowski. Analytical ordered affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/analoaf.html.

[4] Henryk Oryszczyszyn and Krzysztof Prażmowski. Classical configurations in affine planes. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/aff_2.html.

[5] Henryk Oryszczyszyn and Krzysztof Prażmowski. Ordered affine spaces defined in terms of directed parallelity — part I. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/diraf.html.

[6] Henryk Oryszczyszyn and Krzysztof Prażmowski. Parallelity and lines in affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/aff_1.html.

[7] Henryk Oryszczyszyn and Krzysztof Prażmowski. Transformations in affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/transgeo.html.

Received June 12, 1990

Published January 2, 2004