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The Topological Space $\mathcal{E}^2_{\rm T}$. Simple Closed Curves

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Summary. Continuation of [7]. The fact that the unit square is compact is shown in the beginning of the article. Next the notion of simple closed curve is introduced. It is proved that any simple closed curve can be divided into two independent parts which are homeomorphic to unit interval $\mathbb{I}$.

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The articles [9], [10], [11], [2], [3], [8], [5], [4], [1], [6], and [7] provide the notation and terminology for this paper.

In this paper p_1, p_2, q_1, q_2 are points of $\mathcal{E}^2_{\rm T}$.

We now state three propositions:

(1) Suppose $p_1 \neq p_2$ and $p_1 \in \square_{\mathcal{E}^2}$ and $p_2 \in \square_{\mathcal{E}^2}$. Then there exist non empty subsets P_1, P_2 of $\mathcal{E}^2_{\rm T}$ such that P_1 is an arc from p_1 to p_2 and P_2 is an arc from p_1 to p_2 and $\square_{\mathcal{E}^2} = P_1 \cup P_2$ and $P_1 \cap P_2 = \{p_1, p_2\}$.

(2) $\square_{\mathcal{E}^2}$ is compact.

(3) Let Q, P be non empty subsets of $\mathcal{E}^2_{\rm T}$ and f be a map from $(\mathcal{E}^2_{\rm T})\restriction Q$ into $(\mathcal{E}^2_{\rm T})\restriction P$. Suppose f is a homeomorphism and Q is an arc from q_1 to q_2. Let given p_1, p_2. If $p_1 = f(q_1)$ and $p_2 = f(q_2)$, then P is an arc from p_1 to p_2.

Let P be a subset of $\mathcal{E}^2_{\rm T}$. We say that P satisfies conditions of simple closed curve if and only if:

(Def. 1) There exists a map from $\mathcal{E}^2_{\rm T}\restriction\square_{\mathcal{E}^2}$ into $\mathcal{E}^2_{\rm T}\restriction P$ which is a homeomorphism.

We introduce P is a simple closed curve as a synonym of P satisfies conditions of simple closed curve.

One can verify that $\square_{\mathcal{E}^2}$ satisfies conditions of simple closed curve.

Let us observe that there exists a subset of $\mathcal{E}^2_{\rm T}$ which satisfies conditions of simple closed curve.

A simple closed curve is a subset of $\mathcal{E}^2_{\rm T}$ satisfying conditions of simple closed curve.

We now state three propositions:

(4) For every non empty subset P of $\mathcal{E}^2_{\rm T}$ such that P is a simple closed curve there exist p_1, p_2 such that $p_1 \neq p_2$ and $p_1 \in P$ and $p_2 \in P$.

(5) Let P be a non empty subset of $\mathcal{E}^2_{\rm T}$. Then P is a simple closed curve if and only if the following conditions are satisfied:

(i) there exist p_1, p_2 such that $p_1 \neq p_2$ and $p_1 \in P$ and $p_2 \in P$, and

(ii) for all p_1, p_2 such that $p_1 \neq p_2$ and $p_1 \in P$ and $p_2 \in P$ there exist non empty subsets P_1, P_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that P_1 is an arc from p_1 to p_2 and P_2 is an arc from p_1 to p_2 and $P = P_1 \cup P_2$ and $P_1 \cap P_2 = \{p_1, p_2\}$.

(6) Let P be a non empty subset of $\mathcal{E}^2_{\mathrm{T}}$. Then P is a simple closed curve if and only if there exist points p_1, p_2 of $\mathcal{E}^2_{\mathrm{T}}$ and there exist non empty subsets P_1, P_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $p_1 \neq p_2$ and $p_1 \in P$ and $p_2 \in P$ and P_1 is an arc from p_1 to p_2 and P_2 is an arc from p_1 to p_2 and $P = P_1 \cup P_2$ and $P_1 \cap P_2 = \{p_1, p_2\}$.

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