

# The Definition and Basic Properties of Topological Groups

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The articles [15], [7], [20], [21], [9], [4], [6], [3], [12], [11], [18], [19], [10], [13], [22], [8], [2], [1], [16], [23], [14], [5], and [17] provide the notation and terminology for this paper.

## 1. PRELIMINARIES

For simplicity, we follow the rules:  $S$  is a 1-sorted structure,  $R$  is a non empty 1-sorted structure,  $X$  is a subset of  $R$ ,  $T$  is a non empty topological structure, and  $x$  is a set.

Let  $X$  be a set. Observe that there exists a function from  $X$  into  $X$  which is one-to-one and onto.

We now state the proposition

$$(1) \quad \text{rng}(\text{id}_S) = \Omega_S.$$

Let  $R$  be a non empty 1-sorted structure. Note that  $\text{UNKNOWN}(\text{id}_R)$  is one-to-one.

One can prove the following propositions:

$$(2) \quad \text{UNKNOWN}(\text{id}_R) = \text{id}_R.$$

$$(3) \quad (\text{id}_R)^{-1}(X) = X.$$

Let  $S$  be a 1-sorted structure. One can check that there exists a map from  $S$  into  $S$  which is one-to-one and onto.

## 2. ON THE GROUPS

We adopt the following convention:  $H$  denotes a non empty groupoid,  $P, Q, P_1, Q_1$  denote subsets of  $H$ , and  $h$  denotes an element of  $H$ .

We now state three propositions:

$$(4) \quad \text{If } P \subseteq P_1 \text{ and } Q \subseteq Q_1, \text{ then } P \cdot Q \subseteq P_1 \cdot Q_1.$$

$$(5) \quad \text{If } P \subseteq Q, \text{ then } P \cdot h \subseteq Q \cdot h.$$

$$(6) \quad \text{If } P \subseteq Q, \text{ then } h \cdot P \subseteq h \cdot Q.$$

In the sequel  $G$  denotes a group,  $A, B$  denote subsets of  $G$ , and  $a$  denotes an element of  $G$ .

The following propositions are true:

- (7)  $a \in A^{-1}$  iff  $a^{-1} \in A$ .
- (8)  $(A^{-1})^{-1} = A$ .
- (9)  $A \subseteq B$  iff  $A^{-1} \subseteq B^{-1}$ .
- (10)  $\cdot_G^{-1 \circ} A = A^{-1}$ .
- (11)  $\cdot_G^{-1-1}(A) = A^{-1}$ .
- (12)  $\cdot_G^{-1}$  is one-to-one.
- (13)  $\text{rng} \cdot_G^{-1} = \text{the carrier of } G$ .

Let  $G$  be a group. Observe that  $\cdot_G^{-1}$  is one-to-one and onto.  
One can prove the following propositions:

- (14)  $\cdot_G^{-1-1} = \cdot_G^{-1}$ .
- (15) (The multiplication of  $H$ ) $^{\circ}; P, Q:] = P \cdot Q$ .

Let  $G$  be a non empty groupoid and let  $a$  be an element of  $G$ . The functor  $a^*$  yields a map from  $G$  into  $G$  and is defined as follows:

(Def. 1) For every element  $x$  of  $G$  holds  $a^*(x) = a \cdot x$ .

The functor  $\cdot a$  yields a map from  $G$  into  $G$  and is defined as follows:

(Def. 2) For every element  $x$  of  $G$  holds  $(\cdot a)(x) = x \cdot a$ .

Let  $G$  be a group and let  $a$  be an element of  $G$ . Observe that  $a^*$  is one-to-one and onto and  $\cdot a$  is one-to-one and onto.

The following four propositions are true:

- (16)  $(h^*)^{\circ} P = h \cdot P$ .
- (17)  $(\cdot h)^{\circ} P = P \cdot h$ .
- (18) UNKNOWN( $a^*$ ) =  $(a^{-1})^*$ .
- (19) UNKNOWN( $\cdot a$ ) =  $\cdot a^{-1}$ .

### 3. ON THE TOPOLOGICAL SPACES

Let  $T$  be a non empty topological structure. Observe that UNKNOWN( $\text{id}_T$ ) is continuous.

We now state the proposition

- (20)  $\text{id}_T$  is a homeomorphism.

Let  $T$  be a non empty topological space and let  $p$  be a point of  $T$ . Observe that every neighbourhood of  $p$  is non empty.

We now state the proposition

- (21) For every non empty topological space  $T$  and for every point  $p$  of  $T$  holds  $\Omega_T$  is a neighbourhood of  $p$ .

Let  $T$  be a non empty topological space and let  $p$  be a point of  $T$ . Observe that there exists a neighbourhood of  $p$  which is non empty and open.

One can prove the following propositions:

- (22) Let  $S, T$  be non empty topological spaces and  $f$  be a map from  $S$  into  $T$ . Suppose  $f$  is open. Let  $p$  be a point of  $S$  and  $P$  be a neighbourhood of  $p$ . Then there exists an open neighbourhood  $R$  of  $f(p)$  such that  $R \subseteq f^{\circ} P$ .

- (23) Let  $S, T$  be non empty topological spaces and  $f$  be a map from  $S$  into  $T$ . Suppose that for every point  $p$  of  $S$  and for every open neighbourhood  $P$  of  $p$  there exists a neighbourhood  $R$  of  $f(p)$  such that  $R \subseteq f^\circ P$ . Then  $f$  is open.
- (24) Let  $S, T$  be non empty topological structures and  $f$  be a map from  $S$  into  $T$ . Then  $f$  is a homeomorphism if and only if the following conditions are satisfied:
- (i)  $\text{dom } f = \Omega_S$ ,
  - (ii)  $\text{rng } f = \Omega_T$ ,
  - (iii)  $f$  is one-to-one, and
  - (iv) for every subset  $P$  of  $T$  holds  $P$  is closed iff  $f^{-1}(P)$  is closed.
- (25) Let  $S, T$  be non empty topological structures and  $f$  be a map from  $S$  into  $T$ . Then  $f$  is a homeomorphism if and only if the following conditions are satisfied:
- (i)  $\text{dom } f = \Omega_S$ ,
  - (ii)  $\text{rng } f = \Omega_T$ ,
  - (iii)  $f$  is one-to-one, and
  - (iv) for every subset  $P$  of  $S$  holds  $P$  is open iff  $f^\circ P$  is open.
- (26) Let  $S, T$  be non empty topological structures and  $f$  be a map from  $S$  into  $T$ . Then  $f$  is a homeomorphism if and only if the following conditions are satisfied:
- (i)  $\text{dom } f = \Omega_S$ ,
  - (ii)  $\text{rng } f = \Omega_T$ ,
  - (iii)  $f$  is one-to-one, and
  - (iv) for every subset  $P$  of  $T$  holds  $P$  is open iff  $f^{-1}(P)$  is open.
- (27) Let  $S$  be a topological space,  $T$  be a non empty topological space, and  $f$  be a map from  $S$  into  $T$ . Then  $f$  is continuous if and only if for every subset  $P$  of  $T$  holds  $f^{-1}(\text{Int } P) \subseteq \text{Int}(f^{-1}(P))$ .

Let  $T$  be a non empty topological space. Observe that there exists a subset of  $T$  which is non empty and dense.

The following two propositions are true:

- (28) Let  $S, T$  be non empty topological spaces,  $f$  be a map from  $S$  into  $T$ , and  $A$  be a dense subset of  $S$ . If  $f$  is a homeomorphism, then  $f^\circ A$  is dense.
- (29) Let  $S, T$  be non empty topological spaces,  $f$  be a map from  $S$  into  $T$ , and  $A$  be a dense subset of  $T$ . If  $f$  is a homeomorphism, then  $f^{-1}(A)$  is dense.

Let  $S, T$  be non empty topological structures. Observe that every map from  $S$  into  $T$  which is homeomorphism is also onto, one-to-one, continuous, and open.

Let  $T$  be a non empty topological structure. Note that there exists a map from  $T$  into  $T$  which is homeomorphism.

Let  $T$  be a non empty topological structure and let  $f$  be homeomorphism map from  $T$  into  $T$ . One can check that  $\text{UNKNOWN}(f)$  is homeomorphism.

#### 4. THE GROUP OF HOMEOMORPHISMS

Let  $T$  be a non empty topological structure. A map from  $T$  into  $T$  is said to be a homeomorphism of  $T$  if:

- (Def. 3) It is a homeomorphism.

Let  $T$  be a non empty topological structure. Then  $\text{id}_T$  is a homeomorphism of  $T$ .

Let  $T$  be a non empty topological structure. Note that every homeomorphism of  $T$  is homeomorphism.

We now state two propositions:

- (30) For every homeomorphism  $f$  of  $T$  holds  $\text{UNKNOWN}(f)$  is a homeomorphism of  $T$ .
- (31) For all homeomorphisms  $f, g$  of  $T$  holds  $f \cdot g$  is a homeomorphism of  $T$ .

Let  $T$  be a non empty topological structure. The group of homeomorphisms of  $T$  is a strict groupoid and is defined by the conditions (Def. 4).

- (Def. 4)(i)  $x \in$  the carrier of the group of homeomorphisms of  $T$  iff  $x$  is a homeomorphism of  $T$ , and
- (ii) for all homeomorphisms  $f, g$  of  $T$  holds (the multiplication of the group of homeomorphisms of  $T$ )( $f, g$ ) =  $g \cdot f$ .

Let  $T$  be a non empty topological structure. Note that the group of homeomorphisms of  $T$  is non empty.

We now state the proposition

- (32) Let  $f, g$  be homeomorphisms of  $T$  and  $a, b$  be elements of the group of homeomorphisms of  $T$ . If  $f = a$  and  $g = b$ , then  $a \cdot b = g \cdot f$ .

Let  $T$  be a non empty topological structure. One can check that the group of homeomorphisms of  $T$  is group-like and associative.

Next we state two propositions:

- (33)  $\text{id}_T = 1_{\text{the group of homeomorphisms of } T}$ .
- (34) Let  $f$  be a homeomorphism of  $T$  and  $a$  be an element of the group of homeomorphisms of  $T$ . If  $f = a$ , then  $a^{-1} = \text{UNKNOWN}(f)$ .

Let  $T$  be a non empty topological structure. We say that  $T$  is homogeneous if and only if:

- (Def. 5) For all points  $p, q$  of  $T$  there exists a homeomorphism  $f$  of  $T$  such that  $f(p) = q$ .

Let us note that every non empty topological structure which is trivial is also homogeneous.

Let us observe that there exists a topological space which is strict, trivial, and non empty.

One can prove the following two propositions:

- (35) Let  $T$  be a homogeneous non empty topological space. If there exists a point  $p$  of  $T$  such that  $\{p\}$  is closed, then  $T$  is a  $T_1$  space.
- (36) Let  $T$  be a homogeneous non empty topological space. Given a point  $p$  of  $T$  such that let  $A$  be a subset of  $T$ . Suppose  $A$  is open and  $p \in A$ . Then there exists a subset  $B$  of  $T$  such that  $p \in B$  and  $B$  is open and  $\bar{B} \subseteq A$ . Then  $T$  is a  $T_3$  space.

## 5. ON THE TOPOLOGICAL GROUPS

We introduce topological group structures which are extensions of groupoid and topological structure and are systems

$\langle \text{a carrier, a multiplication, a topology} \rangle$ ,

where the carrier is a set, the multiplication is a binary operation on the carrier, and the topology is a family of subsets of the carrier.

Let  $A$  be a non empty set, let  $R$  be a binary operation on  $A$ , and let  $T$  be a family of subsets of  $A$ . Observe that  $\langle A, R, T \rangle$  is non empty.

Let  $x$  be a set, let  $R$  be a binary operation on  $\{x\}$ , and let  $T$  be a family of subsets of  $\{x\}$ . One can check that  $\langle \{x\}, R, T \rangle$  is trivial.

One can check that every non empty groupoid which is trivial is also group-like, associative, and commutative.

Let  $a$  be a set. One can check that  $\{\{a\}\}_{\text{top}}$  is trivial.

Let us note that there exists a topological group structure which is strict and non empty.

One can verify that there exists a non empty topological group structure which is strict, topological space-like, and trivial.

Let  $G$  be a group-like associative non empty topological group structure. Then  $\cdot_G^{-1}$  is a map from  $G$  into  $G$ .

Let  $G$  be a group-like associative non empty topological group structure. We say that  $G$  is inverse-continuous if and only if:

(Def. 6)  $\cdot_G^{-1}$  is continuous.

Let  $G$  be a topological space-like topological group structure. We say that  $G$  is continuous if and only if:

(Def. 7) For every map  $f$  from  $[:G, G:]$  into  $G$  such that  $f =$  the multiplication of  $G$  holds  $f$  is continuous.

Let us note that there exists a topological space-like group-like associative non empty topological group structure which is strict, commutative, trivial, inverse-continuous, and continuous.

A semi topological group is a topological space-like group-like associative non empty topological group structure.

A topological group is an inverse-continuous continuous semi topological group.

We now state several propositions:

- (37) Let  $T$  be a continuous non empty topological space-like topological group structure,  $a, b$  be elements of  $T$ , and  $W$  be a neighbourhood of  $a \cdot b$ . Then there exists an open neighbourhood  $A$  of  $a$  and there exists an open neighbourhood  $B$  of  $b$  such that  $A \cdot B \subseteq W$ .
- (38) Let  $T$  be a topological space-like non empty topological group structure. Suppose that for all elements  $a, b$  of  $T$  and for every neighbourhood  $W$  of  $a \cdot b$  there exists a neighbourhood  $A$  of  $a$  and there exists a neighbourhood  $B$  of  $b$  such that  $A \cdot B \subseteq W$ . Then  $T$  is continuous.
- (39) Let  $T$  be an inverse-continuous semi topological group,  $a$  be an element of  $T$ , and  $W$  be a neighbourhood of  $a^{-1}$ . Then there exists an open neighbourhood  $A$  of  $a$  such that  $A^{-1} \subseteq W$ .
- (40) Let  $T$  be a semi topological group. Suppose that for every element  $a$  of  $T$  and for every neighbourhood  $W$  of  $a^{-1}$  there exists a neighbourhood  $A$  of  $a$  such that  $A^{-1} \subseteq W$ . Then  $T$  is inverse-continuous.
- (41) Let  $T$  be a topological group,  $a, b$  be elements of  $T$ , and  $W$  be a neighbourhood of  $a \cdot b^{-1}$ . Then there exists an open neighbourhood  $A$  of  $a$  and there exists an open neighbourhood  $B$  of  $b$  such that  $A \cdot B^{-1} \subseteq W$ .
- (42) Let  $T$  be a semi topological group. Suppose that for all elements  $a, b$  of  $T$  and for every neighbourhood  $W$  of  $a \cdot b^{-1}$  there exists a neighbourhood  $A$  of  $a$  and there exists a neighbourhood  $B$  of  $b$  such that  $A \cdot B^{-1} \subseteq W$ . Then  $T$  is a topological group.

Let  $G$  be a continuous non empty topological space-like topological group structure and let  $a$  be an element of  $G$ . One can verify that  $a^*$  is continuous and  $\cdot a$  is continuous.

The following two propositions are true:

- (43) For every continuous semi topological group  $G$  and for every element  $a$  of  $G$  holds  $a^*$  is a homeomorphism of  $G$ .
- (44) For every continuous semi topological group  $G$  and for every element  $a$  of  $G$  holds  $\cdot a$  is a homeomorphism of  $G$ .

Let  $G$  be a continuous semi topological group and let  $a$  be an element of  $G$ . Then  $a^*$  is a homeomorphism of  $G$ . Then  $\cdot a$  is a homeomorphism of  $G$ .

Next we state the proposition

- (45) For every inverse-continuous semi topological group  $G$  holds  $\cdot_G^{-1}$  is a homeomorphism of  $G$ .

Let  $G$  be an inverse-continuous semi topological group. Then  $\cdot_G^{-1}$  is a homeomorphism of  $G$ .

One can check that every semi topological group which is continuous is also homogeneous.

Next we state two propositions:

- (46) Let  $G$  be a continuous semi topological group,  $F$  be a closed subset of  $G$ , and  $a$  be an element of  $G$ . Then  $F \cdot a$  is closed.

- (47) Let  $G$  be a continuous semi topological group,  $F$  be a closed subset of  $G$ , and  $a$  be an element of  $G$ . Then  $a \cdot F$  is closed.

Let  $G$  be a continuous semi topological group, let  $F$  be a closed subset of  $G$ , and let  $a$  be an element of  $G$ . One can verify that  $F \cdot a$  is closed and  $a \cdot F$  is closed.

Next we state the proposition

- (48) For every inverse-continuous semi topological group  $G$  and for every closed subset  $F$  of  $G$  holds  $F^{-1}$  is closed.

Let  $G$  be an inverse-continuous semi topological group and let  $F$  be a closed subset of  $G$ . One can check that  $F^{-1}$  is closed.

One can prove the following propositions:

- (49) Let  $G$  be a continuous semi topological group,  $O$  be an open subset of  $G$ , and  $a$  be an element of  $G$ . Then  $O \cdot a$  is open.

- (50) Let  $G$  be a continuous semi topological group,  $O$  be an open subset of  $G$ , and  $a$  be an element of  $G$ . Then  $a \cdot O$  is open.

Let  $G$  be a continuous semi topological group, let  $A$  be an open subset of  $G$ , and let  $a$  be an element of  $G$ . One can check that  $A \cdot a$  is open and  $a \cdot A$  is open.

The following proposition is true

- (51) For every inverse-continuous semi topological group  $G$  and for every open subset  $O$  of  $G$  holds  $O^{-1}$  is open.

Let  $G$  be an inverse-continuous semi topological group and let  $A$  be an open subset of  $G$ . Observe that  $A^{-1}$  is open.

One can prove the following two propositions:

- (52) For every continuous semi topological group  $G$  and for all subsets  $A, O$  of  $G$  such that  $O$  is open holds  $O \cdot A$  is open.

- (53) For every continuous semi topological group  $G$  and for all subsets  $A, O$  of  $G$  such that  $O$  is open holds  $A \cdot O$  is open.

Let  $G$  be a continuous semi topological group, let  $A$  be an open subset of  $G$ , and let  $B$  be a subset of  $G$ . Note that  $A \cdot B$  is open and  $B \cdot A$  is open.

The following propositions are true:

- (54) Let  $G$  be an inverse-continuous semi topological group,  $a$  be a point of  $G$ , and  $A$  be a neighbourhood of  $a$ . Then  $A^{-1}$  is a neighbourhood of  $a^{-1}$ .

- (55) Let  $G$  be a topological group,  $a$  be a point of  $G$ , and  $A$  be a neighbourhood of  $a \cdot a^{-1}$ . Then there exists an open neighbourhood  $B$  of  $a$  such that  $B \cdot B^{-1} \subseteq A$ .

- (56) For every inverse-continuous semi topological group  $G$  and for every dense subset  $A$  of  $G$  holds  $A^{-1}$  is dense.

Let  $G$  be an inverse-continuous semi topological group and let  $A$  be a dense subset of  $G$ . Observe that  $A^{-1}$  is dense.

Next we state two propositions:

- (57) Let  $G$  be a continuous semi topological group,  $A$  be a dense subset of  $G$ , and  $a$  be a point of  $G$ . Then  $a \cdot A$  is dense.
- (58) Let  $G$  be a continuous semi topological group,  $A$  be a dense subset of  $G$ , and  $a$  be a point of  $G$ . Then  $A \cdot a$  is dense.

Let  $G$  be a continuous semi topological group, let  $A$  be a dense subset of  $G$ , and let  $a$  be a point of  $G$ . Observe that  $A \cdot a$  is dense and  $a \cdot A$  is dense.

We now state two propositions:

- (59) Let  $G$  be a topological group,  $B$  be a basis of  $1_G$ , and  $M$  be a dense subset of  $G$ . Then  $\{V \cdot x; V \text{ ranges over subsets of } G, x \text{ ranges over points of } G: V \in B \wedge x \in M\}$  is a basis of  $G$ .
- (60) Every topological group is a  $T_3$  space.

Let us mention that every topological group is  $T_3$ .

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