JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

A Construction of an Abstract Space of Congruence of Vectors[1]

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Summary. In the class of abelian groups a subclass of two-divisible-groups is singled out, and in the latter, a subclass of uniquely-two-divisible-groups. With every such a group a special geometrical structure, more precisely the structure of "congruence of vectors" is correlated. The notion of "affine vector space" (denoted by AffVect) is introduced. This term is defined by means of suitable axiom system. It is proved that every structure of the congruence of vectors determined by a non trivial uniquely two divisible group is a affine vector space.

MML Identifier: TDGROUP.

WWW: http://mizar.org/JFM/Vol2/tdgroup.html

The articles [6], [3], [9], [7], [5], [2], [10], [8], [4], and [1] provide the notation and terminology for this paper.

The following propositions are true:

(2)[1] For every element a of $\mathbb{R}_{\rm G}$ there exists an element b of $\mathbb{R}_{\rm G}$ such that $b+b=a$.

(3) For every element a of $\mathbb{R}_{\rm G}$ such that $a+a=0_{\mathbb{R}_{\rm G}}$ holds $a=0_{\mathbb{R}_{\rm G}}$.

Let I_1 be a non empty loop structure. We say that I_1 is 2-divisible if and only if:

(Def. 1) For every element a of I_1 there exists an element b of I_1 such that $b+b=a$.

Let us mention that $\mathbb{R}_{\rm G}$ is Fanoian and 2-divisible.

Let us note that there exists a non empty loop structure which is strict, Fanoian, 2-divisible, add-associative, right zeroed, right complementable, and Abelian.

A 2-divisible group is a 2-divisible add-associative right zeroed right complementable Abelian non empty loop structure.

A uniquely 2-divisible group is a Fanoian 2-divisible add-associative right zeroed right complementable Abelian non empty loop structure.

One can prove the following proposition

(7)[2] Let A_1 be an add-associative right zeroed right complementable Abelian non empty loop structure. Then A_1 is a uniquely 2-divisible group if and only if the following conditions are satisfied:

(i) for every element a of A_1 there exists an element b of A_1 such that $b+b=a$, and

(ii) for every element a of A_1 such that $a+a=0_{(A_1)}$ holds $a=0_{(A_1)}$.

[1]Supported by RPBP.III-24.C3.

[1] The proposition (1) has been removed.

[2] The propositions (4)–(6) have been removed.

We follow the rules: A_2 denotes a uniquely 2-divisible group and a, b, c, d, a', b', c', p, q denote elements of A_2.

Let A_2 be a non empty loop structure and let a, b be elements of A_2. We introduce $a\#b$ as a synonym of $a+b$.

Let A_2 be a non empty loop structure. The functor $\mathrm{Congr}_{(A_2)}$ yielding a binary relation on [: the carrier of A_2, the carrier of A_2 :] is defined by:

(Def. 4)[3] For all elements a, b, c, d of A_2 holds $\langle\langle a, b\rangle, \langle c, d\rangle\rangle \in \mathrm{Congr}_{(A_2)}$ iff $a\#d = b\#c$.

Let A_2 be a non empty loop structure. The functor $\mathrm{Vectors}(A_2)$ yielding a strict affine structure is defined by:

(Def. 5) $\mathrm{Vectors}(A_2) = \langle$the carrier of A_2, $\mathrm{Congr}_{(A_2)}\rangle$.

Let A_2 be a non empty loop structure. Note that $\mathrm{Vectors}(A_2)$ is non empty.

We now state the proposition

(9)[4] The carrier of $\mathrm{Vectors}(A_2)$ = the carrier of A_2 and the congruence of $\mathrm{Vectors}(A_2) = \mathrm{Congr}_{(A_2)}$.

Let us consider A_2 and let us consider a, b, c, d. The predicate $a,b \Rightarrow c,d$ is defined as follows:

(Def. 6) $\langle\langle a, b\rangle, \langle c, d\rangle\rangle \in$ the congruence of $\mathrm{Vectors}(A_2)$.

Next we state a number of propositions:

(10) $a,b \Rightarrow c,d$ iff $a\#d = b\#c$.

(11) There exist elements a, b of $\mathbb{R}_{\mathrm{G}}$ such that $a \neq b$.

(12) There exists A_2 and there exist a, b such that $a \neq b$.

(13) If $a,b \Rightarrow c,c$, then $a = b$.

(14) If $a,b \Rightarrow p,q$ and $c,d \Rightarrow p,q$, then $a,b \Rightarrow c,d$.

(15) There exists d such that $a,b \Rightarrow c,d$.

(16) If $a,b \Rightarrow a',b'$ and $a,c \Rightarrow a',c'$, then $b,c \Rightarrow b',c'$.

(17) There exists b such that $a,b \Rightarrow b,c$.

(18) If $a,b \Rightarrow b,c$ and $a,b' \Rightarrow b',c$, then $b = b'$.

(19) If $a,b \Rightarrow c,d$, then $a,c \Rightarrow b,d$.

In the sequel A_3 is a non empty affine structure.

Next we state the proposition

(20) Given elements a, b of A_2 such that $a \neq b$. Then there exist elements a, b of $\mathrm{Vectors}(A_2)$ such that $a \neq b$ and for all elements a, b, c of $\mathrm{Vectors}(A_2)$ such that $a,b \upharpoonleft\!\upharpoonright c,c$ holds $a = b$ and for all elements a, b, c, d, p, q of $\mathrm{Vectors}(A_2)$ such that $a,b \upharpoonleft\!\upharpoonright p,q$ and $c,d \upharpoonleft\!\upharpoonright p,q$ holds $a,b \upharpoonleft\!\upharpoonright c,d$ and for all elements a, b, c of $\mathrm{Vectors}(A_2)$ there exists an element d of $\mathrm{Vectors}(A_2)$ such that $a,b \upharpoonleft\!\upharpoonright c,d$ and for all elements a, b, c, a', b', c' of $\mathrm{Vectors}(A_2)$ such that $a,b \upharpoonleft\!\upharpoonright a',b'$ and $a,c \upharpoonleft\!\upharpoonright a',c'$ holds $b,c \upharpoonleft\!\upharpoonright b',c'$ and for all elements a, c of $\mathrm{Vectors}(A_2)$ there exists an element b of $\mathrm{Vectors}(A_2)$ such that $a,b \upharpoonleft\!\upharpoonright b,c$ and for all elements a, b, c, b' of $\mathrm{Vectors}(A_2)$ such that $a,b \upharpoonleft\!\upharpoonright b,c$ and $a,b' \upharpoonleft\!\upharpoonright b',c$ holds $b = b'$ and for all elements a, b, c, d of $\mathrm{Vectors}(A_2)$ such that $a,b \upharpoonleft\!\upharpoonright c,d$ holds $a,c \upharpoonleft\!\upharpoonright b,d$.

Let I_1 be a non empty affine structure. We say that I_1 is space of free vectors-like if and only if the conditions (Def. 8) are satisfied.

[3] The definitions (Def. 2) and (Def. 3) have been removed.

[4] The proposition (8) has been removed.

(Def. 8)[5] For all elements a, b, c of I_1 such that $a,b \upuparrows c,c$ holds $a=b$ and for all elements a, b, c, d, p, q of I_1 such that $a,b \upuparrows p,q$ and $c,d \upuparrows p,q$ holds $a,b \upuparrows c,d$ and for all elements a, b, c of I_1 there exists an element d of I_1 such that $a,b \upuparrows c,d$ and for all elements a, b, c, a', b', c' of I_1 such that $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ holds $b,c \upuparrows b',c'$ and for all elements a, c of I_1 there exists an element b of I_1 such that $a,b \upuparrows b,c$ and for all elements a, b, c, b' of I_1 such that $a,b \upuparrows b,c$ and $a,b' \upuparrows b',c$ holds $b=b'$ and for all elements a, b, c, d of I_1 such that $a,b \upuparrows c,d$ holds $a,c \upuparrows b,d$.

Let us note that there exists a non empty affine structure which is strict, non trivial, and space of free vectors-like.

A space of free vectors is a non trivial space of free vectors-like non empty affine structure.

The following two propositions are true:

(21) Let given A_3. Then there exist elements a, b of A_3 such that $a \neq b$ and for all elements a, b, c of A_3 such that $a,b \upuparrows c,c$ holds $a=b$ and for all elements a, b, c, d, p, q of A_3 such that $a,b \upuparrows p,q$ and $c,d \upuparrows p,q$ holds $a,b \upuparrows c,d$ and for all elements a, b, c of A_3 there exists an element d of A_3 such that $a,b \upuparrows c,d$ and for all elements a, b, c, a', b', c' of A_3 such that $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ holds $b,c \upuparrows b',c'$ and for all elements a, c of A_3 there exists an element b of A_3 such that $a,b \upuparrows b,c$ and for all elements a, b, c, b' of A_3 such that $a,b \upuparrows b,c$ and $a,b' \upuparrows b',c$ holds $b=b'$ and for all elements a, b, c, d of A_3 such that $a,b \upuparrows c,d$ holds $a,c \upuparrows b,d$ if and only if A_3 is a space of free vectors.

(22) If there exist elements a, b of A_2 such that $a \neq b$, then $\mathrm{Vectors}(A_2)$ is a space of free vectors.

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Received May 23, 1990

Published January 2, 2004

[5] The definition (Def. 7) has been removed.