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T_0 Topological Spaces

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The articles [8], [10], [5], [13], [12], [11], [1], [3], [2], [6], [4], [9], and [7] provide the notation and terminology for this paper.

One can prove the following two propositions:

(1) Let A, B be non empty sets and R_1, R_2 be relations between A and B. Suppose that for every element x of A and for every element y of B holds $\langle x, y\rangle \in R_1$ iff $\langle x, y\rangle \in R_2$. Then $R_1 = R_2$.

(2) Let X, Y be non empty sets, f be a function from X into Y, and A be a subset of X. Suppose that for all elements x_1, x_2 of X such that $x_1 \in A$ and $f(x_1) = f(x_2)$ holds $x_2 \in A$. Then $f^{-1}(f^\circ A) = A$.

Let T, S be topological structures. We say that T and S are homeomorphic if and only if:

(Def. 1) There exists a map from T into S which is a homeomorphism.

Let T, S be topological structures and let f be a map from T into S. We say that f is open if and only if:

(Def. 2) For every subset A of T such that A is open holds $f^\circ A$ is open.

Let T be a non empty topological structure. The functor Indiscernibility(T) yields an equivalence relation of the carrier of T and is defined by the condition (Def. 3).

(Def. 3) Let p, q be points of T. Then $\langle p, q\rangle \in$ Indiscernibility(T) if and only if for every subset A of T such that A is open holds $p \in A$ iff $q \in A$.

Let T be a non empty topological structure. The functor $T_{/\,\mathrm{Indiscernibility}\,T}$ yielding a non empty partition of the carrier of T is defined as follows:

(Def. 4) $T_{/\,\mathrm{Indiscernibility}\,T} = \mathrm{Classes}\,\mathrm{Indiscernibility}(T)$.

Let T be a non empty topological space. The functor T_0-reflex(T) yielding a topological space is defined as follows:

(Def. 5) T_0-reflex(T) = the decomposition space of $T_{/\,\mathrm{Indiscernibility}\,T}$.

Let T be a non empty topological space. Note that T_0-reflex(T) is non empty.

Let T be a non empty topological space. The functor T_0-map(T) yields a continuous map from T into T_0-reflex(T) and is defined as follows:

(Def. 6) T_0-map(T) = the projection onto $T_{/\,\mathrm{Indiscernibility}\,T}$.

We now state a number of propositions:

(3) For every non empty topological space T and for every point p of T holds $p \in (T_0\text{-map}(T))(p)$.

(4) For every non empty topological space T holds $\operatorname{dom} T_0\text{-map}(T) =$ the carrier of T and $\operatorname{rng} T_0\text{-map}(T) \subseteq$ the carrier of $T_0\text{-reflex}(T)$.

(5) Let T be a non empty topological space. Then the carrier of $T_0\text{-reflex}(T) = T_{/\,\text{Indiscernibility}\,T}$ and the topology of $T_0\text{-reflex}(T) = \{A; A$ ranges over subsets of $T_{/\,\text{Indiscernibility}\,T}$: $\bigcup A \in$ the topology of $T\}$.

(6) Let T be a non empty topological space and V be a subset of $T_0\text{-reflex}(T)$. Then V is open if and only if $\bigcup V \in$ the topology of T.

(7) Let T be a non empty topological space and C be a set. Then C is a point of $T_0\text{-reflex}(T)$ if and only if there exists a point p of T such that $C = [p]_{\text{Indiscernibility}(T)}$.

(8) For every non empty topological space T and for every point p of T holds $(T_0\text{-map}(T))(p) = [p]_{\text{Indiscernibility}(T)}$.

(9) For every non empty topological space T and for all points p, q of T holds $(T_0\text{-map}(T))(q) = (T_0\text{-map}(T))(p)$ iff $\langle q, p \rangle \in \text{Indiscernibility}(T)$.

(10) Let T be a non empty topological space and A be a subset of T. Suppose A is open. Let p, q be points of T. If $p \in A$ and $(T_0\text{-map}(T))(p) = (T_0\text{-map}(T))(q)$, then $q \in A$.

(11) Let T be a non empty topological space and A be a subset of T. Suppose A is open. Let C be a subset of T. If $C \in T_{/\,\text{Indiscernibility}\,T}$ and C meets A, then $C \subseteq A$.

(12) For every non empty topological space T holds $T_0\text{-map}(T)$ is open.

Let I_1 be a topological structure. We say that I_1 is discernible if and only if the conditions (Def. 7) are satisfied.

(Def. 7)(i) I_1 is empty, or

(ii) for all points x, y of I_1 such that $x \neq y$ there exists a subset V of I_1 such that V is open but $x \in V$ and $y \notin V$ or $y \in V$ and $x \notin V$.

Let us note that there exists a topological space which is discernible and non empty.

A T_0-space is a discernible non empty topological space.

The following propositions are true:

(13) For every non empty topological space T holds $T_0\text{-reflex}(T)$ is a T_0-space.

(14) Let T, S be non empty topological spaces. Given a map h from $T_0\text{-reflex}(S)$ into $T_0\text{-reflex}(T)$ such that h is a homeomorphism and $T_0\text{-map}(T)$ and $h \cdot T_0\text{-map}(S)$ are fiberwise equipotent. Then T and S are homeomorphic.

(15) Let T be a non empty topological space, T_0 be a T_0-space, f be a continuous map from T into T_0, and p, q be points of T. If $\langle p, q \rangle \in \text{Indiscernibility}(T)$, then $f(p) = f(q)$.

(16) Let T be a non empty topological space, T_0 be a T_0-space, f be a continuous map from T into T_0, and p be a point of T. Then $f^\circ([p]_{\text{Indiscernibility}(T)}) = \{f(p)\}$.

(17) Let T be a non empty topological space, T_0 be a T_0-space, and f be a continuous map from T into T_0. Then there exists a continuous map h from $T_0\text{-reflex}(T)$ into T_0 such that $f = h \cdot T_0\text{-map}(T)$.

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