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The Construction and Computation of While-Loop Programs for SCMPDS[1]

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Summary. This article defines two while-loop statements on SCMPDS, i.e. "while<0" and "while>0", which resemble the while-statements of the common high language such as C. We previously presented a number of tricks for computing while-loop statements on SCMFSA, e.g. step-while. However, after inspecting a few realistic examples, we found that they are neither very useful nor of generalization. To cover much more computation cases of while-loop statements, we generalize the computation model of while-loop statements, based on the principle of Hoare's axioms on the verification of programs.

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The articles [22], [6], [19], [23], [5], [7], [14], [21], [2], [15], [16], [20], [17], [4], [13], [8], [1], [11], [9], [10], [12], [3], and [18] provide the notation and terminology for this paper.

1. Preliminaries

In this paper x, a are Int positions and s is a state of SCMPDS.

Next we state the proposition

(1) For every Int position a there exists a natural number i such that $a = \mathrm{intpos}\,i$.

Let t be a state of SCMPDS. The functor Dstate t yielding a state of SCMPDS is defined by the condition (Def. 1).

(Def. 1) Let x be a set. Then

- (i) if $x \in \text{Data-Loc}_{\rm SCM}$, then $(\mathrm{Dstate}\,t)(x) = t(x)$,
- (ii) if $x \in$ the instruction locations of SCMPDS, then $(\mathrm{Dstate}\,t)(x) = \mathrm{goto}\ 0$, and
- (iii) if $x = \mathbf{IC}_{\rm SCMPDS}$, then $(\mathrm{Dstate}\,t)(x) = \mathrm{inspos}\,0$.

The following four propositions are true:

(2) For all states t_1, t_2 of SCMPDS such that $t_1 \restriction \text{Data-Loc}_{\rm SCM} = t_2 \restriction \text{Data-Loc}_{\rm SCM}$ holds $\mathrm{Dstate}\,t_1 = \mathrm{Dstate}\,t_2$.

(3) For every state t of SCMPDS and for every instruction i of SCMPDS such that $\mathrm{InsCode}(i) \in \{0,4,5,6\}$ holds $\mathrm{Dstate}\,t = \mathrm{Dstate}\,\mathrm{Exec}(i,t)$.

(4) $(\mathrm{Dstate}\,s)(a) = s(a)$.

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(5) Let a be an Int position. Then there exists a function f from $\prod$ (the object kind of SCMPDS) into $\mathbb{N}$ such that for every state s of SCMPDS holds

(i) if $s(a) \leq 0$, then $f(s) = 0$, and

(ii) if $s(a) > 0$, then $f(s) = s(a)$.

2. The Construction and Several Basic Properties of "while<0" Program

Let a be an Int position, let i be an integer, and let I be a Program-block. The functor while $<$ $0(a,i,I)$ yields a Program-block and is defined by:

(Def. 2) while $< 0(a,i,I) = ((a,i) >= 0\_\text{goto card} I + 2); I; \text{goto } (-(\text{card} I + 1))$.

Let I be a shiftable Program-block, let a be an Int position, and let i be an integer. One can check that while $< 0(a,i,I)$ is shiftable.

Let I be a No-StopCode Program-block, let a be an Int position, and let i be an integer. Observe that while $< 0(a,i,I)$ is No-StopCode.

We now state several propositions:

(6) For every Int position a and for every integer i and for every Program-block I holds card while $< 0(a,i,I) = \text{card} I + 2$.

(7) Let a be an Int position, i be an integer, m be a natural number, and I be a Program-block. Then $m < \text{card} I + 2$ if and only if inspos $m \in \text{dom}$ while $< 0(a,i,I)$.

(8) Let a be an Int position, i be an integer, and I be a Program-block. Then (while $< 0(a,i,I))(\text{inspos}\, 0) = (a,i) >= 0\_\text{goto card} I + 2$ and (while $< 0(a,i,I))(\text{inspos card} I + 1) = \text{goto } (-(\text{card} I + 1))$.

(9) Let s be a state of SCMPDS, I be a Program-block, a be an Int position, and i be an integer. If $s(\text{DataLoc}(s(a),i)) \geq 0$, then while $< 0(a,i,I)$ is closed on s and while $< 0(a,i,I)$ is halting on s.

(10) Let s be a state of SCMPDS, I be a Program-block, a, c be Int positions, and i be an integer. If $s(\text{DataLoc}(s(a),i)) \geq 0$, then IExec(while $< 0(a,i,I),s) = s+\cdot$ Start-At(inspos card $I + 2$).

(11) Let s be a state of SCMPDS, I be a Program-block, a be an Int position, and i be an integer. If $s(\text{DataLoc}(s(a),i)) \geq 0$, then $\mathbf{IC}_{\text{IExec}(\text{while}<0(a,i,I),s)} = \text{inspos card} I + 2$.

(12) Let s be a state of SCMPDS, I be a Program-block, a, b be Int positions, and i be an integer. If $s(\text{DataLoc}(s(a),i)) \geq 0$, then (IExec(while $< 0(a,i,I),s))(b) = s(b)$.

In this article we present several logical schemes. The scheme *WhileLHalt* deals with a unary functor $\mathcal{F}$ yielding a natural number, a state $\mathcal{A}$ of SCMPDS, a No-StopCode shiftable Program-block $\mathcal{B}$, an Int position $\mathcal{C}$, an integer $\mathcal{D}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{F}(\mathcal{A}) = \mathcal{F}(\mathcal{A})$ or $\mathcal{P}[\mathcal{A}]$ but while $< 0(\mathcal{C},\mathcal{D},\mathcal{B})$ is closed on $\mathcal{A}$ but while $< 0(\mathcal{C},\mathcal{D},\mathcal{B})$ is halting on $\mathcal{A}$

provided the following conditions are met:

- card $\mathcal{B} > 0$,
- For every state t of SCMPDS such that $\mathcal{P}[\text{Dstate}\, t]$ and $\mathcal{F}(\text{Dstate}\, t) = 0$ holds $t(\text{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) \geq 0$,
- $\mathcal{P}[\text{Dstate}\, \mathcal{A}]$, and
- Let t be a state of SCMPDS. Suppose $\mathcal{P}[\text{Dstate}\, t]$ and $t(\mathcal{C}) = \mathcal{A}(\mathcal{C})$ and $t(\text{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) < 0$. Then $(\text{IExec}(\mathcal{B},t))(\mathcal{C}) = t(\mathcal{C})$ and $\mathcal{B}$ is closed on t and $\mathcal{B}$ is halting on t and $\mathcal{F}(\text{Dstate IExec}(\mathcal{B},t)) < \mathcal{F}(\text{Dstate}\, t)$ and $\mathcal{P}[\text{Dstate IExec}(\mathcal{B},t)]$.

The scheme *WhileLExec* deals with a unary functor $\mathcal{F}$ yielding a natural number, a state $\mathcal{A}$ of SCMPDS, a No-StopCode shiftable Program-block $\mathcal{B}$, an Int position $\mathcal{C}$, an integer $\mathcal{D}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{F}(\mathcal{A}) = \mathcal{F}(\mathcal{A})$ or $\mathcal{P}[\mathcal{A}]$ but IExec(while $< 0(\mathcal{C},\mathcal{D},\mathcal{B}),\mathcal{A})$ = IExec(while $< 0(\mathcal{C},\mathcal{D},\mathcal{B}),\text{IExec}(\mathcal{B},\mathcal{A}))$

provided the parameters meet the following conditions:

- $\mathrm{card}\,\mathcal{B} > 0$,
- $\mathcal{A}(\mathrm{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) < 0$,
- For every state t of SCMPDS such that $\mathcal{P}[\mathrm{Dstate}\,t]$ and $\mathcal{F}(\mathrm{Dstate}\,t) = 0$ holds $t(\mathrm{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) \geq 0$,
- $\mathcal{P}[\mathrm{Dstate}\,\mathcal{A}]$, and
- Let t be a state of SCMPDS. Suppose $\mathcal{P}[\mathrm{Dstate}\,t]$ and $t(\mathcal{C}) = \mathcal{A}(\mathcal{C})$ and $t(\mathrm{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) < 0$. Then $(\mathrm{IExec}(\mathcal{B},t))(\mathcal{C}) = t(\mathcal{C})$ and $\mathcal{B}$ is closed on t and $\mathcal{B}$ is halting on t and $\mathcal{F}(\mathrm{Dstate}\,\mathrm{IExec}(\mathcal{B},t)) < \mathcal{F}(\mathrm{Dstate}\,t)$ and $\mathcal{P}[\mathrm{Dstate}\,\mathrm{IExec}(\mathcal{B},t)]$.

Next we state four propositions:

(13) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i be an integer, X be a set, and f be a function from $\prod$(the object kind of SCMPDS) into $\mathbb{N}$. Suppose that

(i) $\mathrm{card}\,I > 0$,

(ii) for every state t of SCMPDS such that $f(\mathrm{Dstate}\,t) = 0$ holds $t(\mathrm{DataLoc}(s(a),i)) \geq 0$, and

(iii) for every state t of SCMPDS such that for every Int position x such that $x \in X$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) < 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and $f(\mathrm{Dstate}\,\mathrm{IExec}(I,t)) < f(\mathrm{Dstate}\,t)$ and I is closed on t and halting on t and for every Int position x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then $\mathrm{while}<0(a,i,I)$ is closed on s and $\mathrm{while}<0(a,i,I)$ is halting on s.

(14) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i be an integer, X be a set, and f be a function from $\prod$(the object kind of SCMPDS) into $\mathbb{N}$. Suppose that

(i) $\mathrm{card}\,I > 0$,

(ii) $s(\mathrm{DataLoc}(s(a),i)) < 0$,

(iii) for every state t of SCMPDS such that $f(\mathrm{Dstate}\,t) = 0$ holds $t(\mathrm{DataLoc}(s(a),i)) \geq 0$, and

(iv) for every state t of SCMPDS such that for every Int position x such that $x \in X$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) < 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and I is closed on t and halting on t and $f(\mathrm{Dstate}\,\mathrm{IExec}(I,t)) < f(\mathrm{Dstate}\,t)$ and for every Int position x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then $\mathrm{IExec}(\mathrm{while}<0(a,i,I),s) = \mathrm{IExec}(\mathrm{while}<0(a,i,I),\mathrm{IExec}(I,s))$.

(15) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i be an integer, and X be a set. Suppose that

(i) $\mathrm{card}\,I > 0$, and

(ii) for every state t of SCMPDS such that for every Int position x such that $x \in X$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) < 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and $(\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i)) > t(\mathrm{DataLoc}(s(a),i))$ and I is closed on t and halting on t and for every Int position x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then $\mathrm{while}<0(a,i,I)$ is closed on s and $\mathrm{while}<0(a,i,I)$ is halting on s.

(16) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i be an integer, and X be a set. Suppose that

(i) $s(\mathrm{DataLoc}(s(a),i)) < 0$,

(ii) $\mathrm{card}\,I > 0$, and

(iii) for every state t of SCMPDS such that for every Int position x such that $x \in X$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) < 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and $(\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i)) > t(\mathrm{DataLoc}(s(a),i))$ and I is closed on t and halting on t and for every Int position x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then $\mathrm{IExec}(\mathrm{while}<0(a,i,I),s) = \mathrm{IExec}(\mathrm{while}<0(a,i,I),\mathrm{IExec}(I,s))$.

3. THE CONSTRUCTION AND SEVERAL BASIC PROPERTIES OF “WHILE>0” PROGRAM

Let a be an Int position, let i be an integer, and let I be a Program-block. The functor while $>0(a,i,I)$ yields a Program-block and is defined by:

(Def. 3) while $>0(a,i,I) = ((a,i) <= 0\_\text{goto card}\, I+2); I;$ goto $(-(\text{card}\, I+1))$.

Let I be a shiftable Program-block, let a be an Int position, and let i be an integer. Observe that while $>0(a,i,I)$ is shiftable.

Let I be a No-StopCode Program-block, let a be an Int position, and let i be an integer. One can verify that while $>0(a,i,I)$ is No-StopCode.

One can prove the following propositions:

(17) For every Int position a and for every integer i and for every Program-block I holds card while $>0(a,i,I) = \text{card}\, I+2$.

(18) Let a be an Int position, i be an integer, m be a natural number, and I be a Program-block. Then $m < \text{card}\, I+2$ if and only if inspos $m \in$ dom while $>0(a,i,I)$.

(19) Let a be an Int position, i be an integer, and I be a Program-block. Then (while $>0(a,i,I))(\text{inspos}\, 0) = (a,i) <= 0\_\text{goto card}\, I+2$ and (while $>0(a,i,I))(\text{inspos card}\, I+1) =$ goto $(-(\text{card}\, I+1))$.

(20) Let s be a state of SCMPDS, I be a Program-block, a be an Int position, and i be an integer. If $s(\text{DataLoc}(s(a),i)) \leq 0$, then while $>0(a,i,I)$ is closed on s and while $>0(a,i,I)$ is halting on s.

(21) Let s be a state of SCMPDS, I be a Program-block, a, c be Int positions, and i be an integer. If $s(\text{DataLoc}(s(a),i)) \leq 0$, then IExec(while $>0(a,i,I),s) = s+\cdot$ Start-At(inspos card $I+2$).

(22) Let s be a state of SCMPDS, I be a Program-block, a be an Int position, and i be an integer. If $s(\text{DataLoc}(s(a),i)) \leq 0$, then $\mathbf{IC}_{\text{IExec}(\text{while}>0(a,i,I),s)} = \text{inspos card}\, I+2$.

(23) Let s be a state of SCMPDS, I be a Program-block, a, b be Int positions, and i be an integer. If $s(\text{DataLoc}(s(a),i)) \leq 0$, then (IExec(while $>0(a,i,I),s))(b) = s(b)$.

Now we present two schemes. The scheme *WhileGHalt* deals with a unary functor $\mathcal{F}$ yielding a natural number, a state $\mathcal{A}$ of SCMPDS, a No-StopCode shiftable Program-block $\mathcal{B}$, an Int position $\mathcal{C}$, an integer $\mathcal{D}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{F}(\mathcal{A}) = \mathcal{F}(\mathcal{A})$ or $\mathcal{P}[\mathcal{A}]$ but while $>0(\mathcal{C},\mathcal{D},\mathcal{B})$ is closed on $\mathcal{A}$ but while $>0(\mathcal{C},\mathcal{D},\mathcal{B})$ is halting on $\mathcal{A}$

provided the following requirements are met:

- card $\mathcal{B} > 0$,
- For every state t of SCMPDS such that $\mathcal{P}[\text{Dstate}\, t]$ and $\mathcal{F}(\text{Dstate}\, t) = 0$ holds $t(\text{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) \leq 0$,
- $\mathcal{P}[\text{Dstate}\, \mathcal{A}]$, and
- Let t be a state of SCMPDS. Suppose $\mathcal{P}[\text{Dstate}\, t]$ and $t(\mathcal{C}) = \mathcal{A}(\mathcal{C})$ and $t(\text{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) > 0$. Then $(\text{IExec}(\mathcal{B},t))(\mathcal{C}) = t(\mathcal{C})$ and $\mathcal{B}$ is closed on t and $\mathcal{B}$ is halting on t and $\mathcal{F}(\text{Dstate IExec}(\mathcal{B},t)) < \mathcal{F}(\text{Dstate}\, t)$ and $\mathcal{P}[\text{Dstate IExec}(\mathcal{B},t)]$.

The scheme *WhileGExec* deals with a unary functor $\mathcal{F}$ yielding a natural number, a state $\mathcal{A}$ of SCMPDS, a No-StopCode shiftable Program-block $\mathcal{B}$, an Int position $\mathcal{C}$, an integer $\mathcal{D}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{F}(\mathcal{A}) = \mathcal{F}(\mathcal{A})$ or $\mathcal{P}[\mathcal{A}]$ but IExec(while $>0(\mathcal{C},\mathcal{D},\mathcal{B}),\mathcal{A}) =$ IExec(while $>0(\mathcal{C},\mathcal{D},\mathcal{B}),\text{IExec}(\mathcal{B},\mathcal{A}))$

provided the parameters have the following properties:

- card $\mathcal{B} > 0$,
- $\mathcal{A}(\text{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) > 0$,
- For every state t of SCMPDS such that $\mathcal{P}[\text{Dstate}\, t]$ and $\mathcal{F}(\text{Dstate}\, t) = 0$ holds $t(\text{DataLoc}(\mathcal{A}(\mathcal{C}),\mathcal{D})) \leq 0$,
- $\mathcal{P}[\text{Dstate}\, \mathcal{A}]$, and

- Let t be a state of SCMPDS. Suppose $\mathcal{P}[\mathrm{Dstate}\,t]$ and $t(\mathcal{C}) = \mathcal{A}(\mathcal{C})$ and $t(\mathrm{DataLoc}(\mathcal{A}(\mathcal{C}), \mathcal{D})) > 0$. Then $(\mathrm{IExec}(\mathcal{B},t))(\mathcal{C}) = t(\mathcal{C})$ and $\mathcal{B}$ is closed on t and $\mathcal{B}$ is halting on t and $\mathcal{F}(\mathrm{Dstate}\,\mathrm{IExec}(\mathcal{B},t)) < \mathcal{F}(\mathrm{Dstate}\,t)$ and $\mathcal{P}[\mathrm{Dstate}\,\mathrm{IExec}(\mathcal{B},t)]$.

The following propositions are true:

(24) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i, c be integers, X, Y be sets, and f be a function from $\prod$(the object kind of SCMPDS) into $\mathbb{N}$. Suppose that

(i) $\mathrm{card}\,I > 0$,

(ii) for every state t of SCMPDS such that $f(\mathrm{Dstate}\,t) = 0$ holds $t(\mathrm{DataLoc}(s(a),i)) \leq 0$,

(iii) for every x such that $x \in X$ holds $s(x) \geq c + s(\mathrm{DataLoc}(s(a),i))$, and

(iv) for every state t of SCMPDS such that for every x such that $x \in X$ holds $t(x) \geq c + t(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in Y$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) > 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and I is closed on t and halting on t and $f(\mathrm{Dstate}\,\mathrm{IExec}(I,t)) < f(\mathrm{Dstate}\,t)$ and for every x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) \geq c + (\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in Y$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then while $> 0(a,i,I)$ is closed on s and while $> 0(a,i,I)$ is halting on s.

(25) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i, c be integers, X, Y be sets, and f be a function from $\prod$(the object kind of SCMPDS) into $\mathbb{N}$. Suppose that

(i) $s(\mathrm{DataLoc}(s(a),i)) > 0$,

(ii) $\mathrm{card}\,I > 0$,

(iii) for every state t of SCMPDS such that $f(\mathrm{Dstate}\,t) = 0$ holds $t(\mathrm{DataLoc}(s(a),i)) \leq 0$,

(iv) for every x such that $x \in X$ holds $s(x) \geq c + s(\mathrm{DataLoc}(s(a),i))$, and

(v) for every state t of SCMPDS such that for every x such that $x \in X$ holds $t(x) \geq c + t(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in Y$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) > 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and I is closed on t and halting on t and $f(\mathrm{Dstate}\,\mathrm{IExec}(I,t)) < f(\mathrm{Dstate}\,t)$ and for every x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) \geq c + (\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in Y$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then $\mathrm{IExec}(\text{while} > 0(a,i,I),s) = \mathrm{IExec}(\text{while} > 0(a,i,I),\mathrm{IExec}(I,s))$.

(26) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i be an integer, X be a set, and f be a function from $\prod$(the object kind of SCMPDS) into $\mathbb{N}$. Suppose that

(i) $\mathrm{card}\,I > 0$,

(ii) for every state t of SCMPDS such that $f(\mathrm{Dstate}\,t) = 0$ holds $t(\mathrm{DataLoc}(s(a),i)) \leq 0$, and

(iii) for every state t of SCMPDS such that for every x such that $x \in X$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) > 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and I is closed on t and halting on t and $f(\mathrm{Dstate}\,\mathrm{IExec}(I,t)) < f(\mathrm{Dstate}\,t)$ and for every x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then while $> 0(a,i,I)$ is closed on s and while $> 0(a,i,I)$ is halting on s and if $s(\mathrm{DataLoc}(s(a),i)) > 0$, then $\mathrm{IExec}(\text{while} > 0(a,i,I),s) = \mathrm{IExec}(\text{while} > 0(a,i,I),\mathrm{IExec}(I,s))$.

(27) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i, c be integers, and X, Y be sets. Suppose that

(i) $\mathrm{card}\,I > 0$,

(ii) for every x such that $x \in X$ holds $s(x) \geq c + s(\mathrm{DataLoc}(s(a),i))$, and

(iii) for every state t of SCMPDS such that for every x such that $x \in X$ holds $t(x) \geq c + t(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in Y$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) > 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and I is closed on t and halting on t and $(\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i)) < t(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) \geq c + (\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in Y$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then while $> 0(a,i,I)$ is closed on s and while $> 0(a,i,I)$ is halting on s and if $s(\mathrm{DataLoc}(s(a),i)) > 0$, then $\mathrm{IExec}(\text{while} > 0(a,i,I),s) = \mathrm{IExec}(\text{while} > 0(a,i,I),\mathrm{IExec}(I,s))$.

(28) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i be an integer, and X be a set. Suppose that

(i) $\mathrm{card}\, I > 0$, and

(ii) for every state t of SCMPDS such that for every x such that $x \in X$ holds $t(x) = s(x)$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) > 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and I is closed on t and halting on t and $(\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i)) < t(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) = t(x)$.

Then while $> 0(a,i,I)$ is closed on s and while $> 0(a,i,I)$ is halting on s and if $s(\mathrm{DataLoc}(s(a),i)) > 0$, then $\mathrm{IExec}(\text{while} > 0(a,i,I),s) = \mathrm{IExec}(\text{while} > 0(a,i,I),\mathrm{IExec}(I,s))$.

(29) Let s be a state of SCMPDS, I be a No-StopCode shiftable Program-block, a be an Int position, i, c be integers, and X be a set. Suppose that

(i) $\mathrm{card}\, I > 0$,

(ii) for every x such that $x \in X$ holds $s(x) \geq c + s(\mathrm{DataLoc}(s(a),i))$, and

(iii) for every state t of SCMPDS such that for every x such that $x \in X$ holds $t(x) \geq c + t(\mathrm{DataLoc}(s(a),i))$ and $t(a) = s(a)$ and $t(\mathrm{DataLoc}(s(a),i)) > 0$ holds $(\mathrm{IExec}(I,t))(a) = t(a)$ and I is closed on t and halting on t and $(\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i)) < t(\mathrm{DataLoc}(s(a),i))$ and for every x such that $x \in X$ holds $(\mathrm{IExec}(I,t))(x) \geq c + (\mathrm{IExec}(I,t))(\mathrm{DataLoc}(s(a),i))$.

Then while $> 0(a,i,I)$ is closed on s and while $> 0(a,i,I)$ is halting on s and if $s(\mathrm{DataLoc}(s(a),i)) > 0$, then $\mathrm{IExec}(\text{while} > 0(a,i,I),s) = \mathrm{IExec}(\text{while} > 0(a,i,I),\mathrm{IExec}(I,s))$.

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