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A Small Computer Model with Push-Down Stack[1]

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Summary. The SCMFSA computer can prove the correctness of many algorithms. Unfortunately, it cannot prove the correctness of recursive algorithms. For this reason, this article improves the SCMFSA computer and presents a Small Computer Model with Push-Down Stack (called SCMPDS for short). In addition to conventional arithmetic and "goto" instructions, we increase two new instructions such as "return" and "save instruction-counter" in order to be able to design recursive programs.

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WWW: `http://mizar.org/JFM/Vol11/scmpds_1.html`

The articles [13], [12], [6], [20], [21], [4], [5], [11], [14], [16], [2], [17], [1], [3], [15], [19], [7], [8], [9], [10], and [18] provide the notation and terminology for this paper.

1. Preliminaries

For simplicity, we use the following convention: x_1, x_2, x_3, x_4, x_5 denote sets, i, j, k denote natural numbers, I denotes an element of $\mathbb{Z}_{14}$, i_1 denotes an element of $\text{Instr-Loc}_{\text{SCM}}$, d_1, d_2 denote elements of $\text{Data-Loc}_{\text{SCM}}$, and k_1, k_2 denote integers.

Let x_1, x_2, x_3, x_4 be sets. The functor $\langle x_1, x_2, x_3, x_4 \rangle$ yielding a set is defined by:

(Def. 1) $\langle x_1,x_2,x_3,x_4\rangle = \langle x_1,x_2,x_3\rangle \frown \langle x_4\rangle$.

Let x_5 be a set. The functor $\langle x_1,x_2,x_3,x_4,x_5\rangle$ yields a set and is defined by:

(Def. 2) $\langle x_1,x_2,x_3,x_4,x_5\rangle = \langle x_1,x_2,x_3\rangle \frown \langle x_4,x_5\rangle$.

Let x_1, x_2, x_3, x_4 be sets. Observe that $\langle x_1,x_2,x_3,x_4\rangle$ is function-like and relation-like. Let x_5 be a set. Note that $\langle x_1,x_2,x_3,x_4,x_5\rangle$ is function-like and relation-like.

Let x_1, x_2, x_3, x_4 be sets. Note that $\langle x_1,x_2,x_3,x_4\rangle$ is finite sequence-like. Let x_5 be a set. Observe that $\langle x_1,x_2,x_3,x_4,x_5\rangle$ is finite sequence-like.

Let D be a non empty set and let x_1, x_2, x_3, x_4 be elements of D. Then $\langle x_1,x_2,x_3,x_4\rangle$ is a finite sequence of elements of D.

Let D be a non empty set and let x_1, x_2, x_3, x_4, x_5 be elements of D. Then $\langle x_1,x_2,x_3,x_4,x_5\rangle$ is a finite sequence of elements of D.

One can prove the following propositions:

(1) $\langle x_1,x_2,x_3,x_4\rangle = \langle x_1,x_2,x_3\rangle \frown \langle x_4\rangle$ and $\langle x_1,x_2,x_3,x_4\rangle = \langle x_1,x_2\rangle \frown \langle x_3,x_4\rangle$ and $\langle x_1,x_2,x_3,x_4\rangle = \langle x_1\rangle \frown \langle x_2,x_3,x_4\rangle$ and $\langle x_1,x_2,x_3,x_4\rangle = \langle x_1\rangle \frown \langle x_2\rangle \frown \langle x_3\rangle \frown \langle x_4\rangle$.

(2) $\langle x_1,x_2,x_3,x_4,x_5\rangle = \langle x_1,x_2,x_3\rangle \frown \langle x_4,x_5\rangle$ and $\langle x_1,x_2,x_3,x_4,x_5\rangle = \langle x_1,x_2,x_3,x_4\rangle \frown \langle x_5\rangle$ and $\langle x_1,x_2,x_3,x_4,x_5\rangle = \langle x_1\rangle \frown \langle x_2\rangle \frown \langle x_3\rangle \frown \langle x_4\rangle \frown \langle x_5\rangle$ and $\langle x_1,x_2,x_3,x_4,x_5\rangle = \langle x_1,x_2\rangle \frown \langle x_3,x_4,x_5\rangle$ and $\langle x_1,x_2,x_3,x_4,x_5\rangle = \langle x_1\rangle \frown \langle x_2,x_3,x_4,x_5\rangle$.

[1]This work was done while the author visited Shinshu University March–April 1999.

We follow the rules: N_1 denotes a non empty set, y_1, y_2, y_3, y_4, y_5 denote elements of N_1, and p denotes a finite sequence.

Next we state several propositions:

(3) $p = \langle x_1, x_2, x_3, x_4 \rangle$ iff $\mathrm{len}\, p = 4$ and $p(1) = x_1$ and $p(2) = x_2$ and $p(3) = x_3$ and $p(4) = x_4$.

(4) $\mathrm{dom}\langle x_1, x_2, x_3, x_4 \rangle = \mathrm{Seg}\, 4$.

(5) $p = \langle x_1, x_2, x_3, x_4, x_5 \rangle$ iff $\mathrm{len}\, p = 5$ and $p(1) = x_1$ and $p(2) = x_2$ and $p(3) = x_3$ and $p(4) = x_4$ and $p(5) = x_5$.

(6) $\mathrm{dom}\langle x_1, x_2, x_3, x_4, x_5 \rangle = \mathrm{Seg}\, 5$.

(7) $\langle y_1, y_2, y_3, y_4 \rangle_1 = y_1$ and $\langle y_1, y_2, y_3, y_4 \rangle_2 = y_2$ and $\langle y_1, y_2, y_3, y_4 \rangle_3 = y_3$ and $\langle y_1, y_2, y_3, y_4 \rangle_4 = y_4$.

(8) $\langle y_1, y_2, y_3, y_4, y_5 \rangle_1 = y_1$ and $\langle y_1, y_2, y_3, y_4, y_5 \rangle_2 = y_2$ and $\langle y_1, y_2, y_3, y_4, y_5 \rangle_3 = y_3$ and $\langle y_1, y_2, y_3, y_4, y_5 \rangle_4 = y_4$ and $\langle y_1, y_2, y_3, y_4, y_5 \rangle_5 = y_5$.

(9) For every integer k holds $k \in \bigcup\{\mathbb{Z}\} \cup \mathbb{N}$.

(10) For every integer k holds $k \in \text{Data-Loc}_{\rm SCM} \cup \mathbb{Z}$.

(11) For every element d of $\text{Data-Loc}_{\rm SCM}$ holds $d \in \text{Data-Loc}_{\rm SCM} \cup \mathbb{Z}$.

2. The Construction of SCM with Push-Down Stack

The subset SCMPDS-Instr of $[: \mathbb{Z}_{14}, (\bigcup\{\mathbb{Z}\} \cup \mathbb{N})^* :]$ is defined by the condition (Def. 3).

(Def. 3) $\text{SCMPDS-Instr} = \{\langle 0, \langle l \rangle \rangle : l$ ranges over elements of $\mathbb{Z}\} \cup \{\langle 1, \langle s_1 \rangle \rangle : s_1$ ranges over elements of $\text{Data-Loc}_{\rm SCM}\} \cup \{\langle I, \langle v, c \rangle \rangle; I$ ranges over elements of $\mathbb{Z}_{14}$, v ranges over elements of $\text{Data-Loc}_{\rm SCM}$, c ranges over elements of $\mathbb{Z}$: $I \in \{2, 3\}\} \cup \{\langle I, \langle v, c_1, c_2 \rangle \rangle; I$ ranges over elements of $\mathbb{Z}_{14}$, v ranges over elements of $\text{Data-Loc}_{\rm SCM}$, c_1 ranges over elements of $\mathbb{Z}$, c_2 ranges over elements of $\mathbb{Z}$: $I \in \{4, 5, 6, 7, 8\}\} \cup \{\langle I, \langle v_1, v_2, c_1, c_2 \rangle \rangle; I$ ranges over elements of $\mathbb{Z}_{14}$, v_1 ranges over elements of $\text{Data-Loc}_{\rm SCM}$, v_2 ranges over elements of $\text{Data-Loc}_{\rm SCM}$, c_1 ranges over elements of $\mathbb{Z}$, c_2 ranges over elements of $\mathbb{Z}$: $I \in \{9, 10, 11, 12, 13\}\}$.

We now state the proposition

(13)[1] $\langle 0, \langle 0 \rangle \rangle \in \text{SCMPDS-Instr}$.

One can check that SCMPDS-Instr is non empty.

We now state three propositions:

(14) $k = 0$ or there exists j such that $k = 2 \cdot j + 1$ or there exists j such that $k = 2 \cdot j + 2$.

(15) If $k = 0$, then it is not true that there exists j such that $k = 2 \cdot j + 1$ and it is not true that there exists j such that $k = 2 \cdot j + 2$.

(16)(i) If there exists j such that $k = 2 \cdot j + 1$, then $k \neq 0$ and it is not true that there exists j such that $k = 2 \cdot j + 2$, and

(ii) if there exists j such that $k = 2 \cdot j + 2$, then $k \neq 0$ and it is not true that there exists j such that $k = 2 \cdot j + 1$.

The function SCMPDS-OK from $\mathbb{N}$ into $\{\mathbb{Z}\} \cup \{\text{SCMPDS-Instr}, \text{Instr-Loc}_{\rm SCM}\}$ is defined by:

(Def. 4) $(\text{SCMPDS-OK})(0) = \text{Instr-Loc}_{\rm SCM}$ and for every natural number k holds $(\text{SCMPDS-OK})(2 \cdot k + 1) = \mathbb{Z}$ and $(\text{SCMPDS-OK})(2 \cdot k + 2) = \text{SCMPDS-Instr}$.

[1] The proposition (12) has been removed.

A SCMPDS-State is an element of $\prod$SCMPDS-OK.

We now state several propositions:

(17) $\text{Instr-Loc}_{\text{SCM}} \neq$ SCMPDS-Instr and SCMPDS-Instr $\neq \mathbb{Z}$.

(18) $(\text{SCMPDS-OK})(i) = \text{Instr-Loc}_{\text{SCM}}$ iff $i = 0$.

(19) $(\text{SCMPDS-OK})(i) = \mathbb{Z}$ iff there exists k such that $i = 2 \cdot k + 1$.

(20) $(\text{SCMPDS-OK})(i) = \text{SCMPDS-Instr}$ iff there exists k such that $i = 2 \cdot k + 2$.

(21) $(\text{SCMPDS-OK})(d_1) = \mathbb{Z}$.

(22) $(\text{SCMPDS-OK})(i_1) = \text{SCMPDS-Instr}$.

(23) $\pi_0 \prod \text{SCMPDS-OK} = \text{Instr-Loc}_{\text{SCM}}$.

(24) $\pi_{d_1} \prod \text{SCMPDS-OK} = \mathbb{Z}$.

(25) $\pi_{i_1} \prod \text{SCMPDS-OK} = \text{SCMPDS-Instr}$.

Let s be a SCMPDS-State. The functor $\mathbf{IC}_s$ yielding an element of $\text{Instr-Loc}_{\text{SCM}}$ is defined as follows:

(Def. 5) $\mathbf{IC}_s = s(0)$.

Let s be a SCMPDS-State and let u be an element of $\text{Instr-Loc}_{\text{SCM}}$. The functor $\text{Chg}_{\text{SCM}}(s,u)$ yielding a SCMPDS-State is defined as follows:

(Def. 6) $\text{Chg}_{\text{SCM}}(s,u) = s{+\cdot}(0 \mapsto u)$.

We now state three propositions:

(26) For every SCMPDS-State s and for every element u of $\text{Instr-Loc}_{\text{SCM}}$ holds $(\text{Chg}_{\text{SCM}}(s,u))(0) = u$.

(27) For every SCMPDS-State s and for every element u of $\text{Instr-Loc}_{\text{SCM}}$ and for every element m_1 of $\text{Data-Loc}_{\text{SCM}}$ holds $(\text{Chg}_{\text{SCM}}(s,u))(m_1) = s(m_1)$.

(28) For every SCMPDS-State s and for all elements u, v of $\text{Instr-Loc}_{\text{SCM}}$ holds $(\text{Chg}_{\text{SCM}}(s,u))(v) = s(v)$.

Let s be a SCMPDS-State, let t be an element of $\text{Data-Loc}_{\text{SCM}}$, and let u be an integer. The functor $\text{Chg}_{\text{SCM}}(s,t,u)$ yielding a SCMPDS-State is defined as follows:

(Def. 7) $\text{Chg}_{\text{SCM}}(s,t,u) = s{+\cdot}(t \mapsto u)$.

One can prove the following four propositions:

(29) For every SCMPDS-State s and for every element t of $\text{Data-Loc}_{\text{SCM}}$ and for every integer u holds $(\text{Chg}_{\text{SCM}}(s,t,u))(0) = s(0)$.

(30) For every SCMPDS-State s and for every element t of $\text{Data-Loc}_{\text{SCM}}$ and for every integer u holds $(\text{Chg}_{\text{SCM}}(s,t,u))(t) = u$.

(31) Let s be a SCMPDS-State, t be an element of $\text{Data-Loc}_{\text{SCM}}$, u be an integer, and m_1 be an element of $\text{Data-Loc}_{\text{SCM}}$. If $m_1 \neq t$, then $(\text{Chg}_{\text{SCM}}(s,t,u))(m_1) = s(m_1)$.

(32) Let s be a SCMPDS-State, t be an element of $\text{Data-Loc}_{\text{SCM}}$, u be an integer, and v be an element of $\text{Instr-Loc}_{\text{SCM}}$. Then $(\text{Chg}_{\text{SCM}}(s,t,u))(v) = s(v)$.

Let s be a SCMPDS-State and let a be an element of $\text{Data-Loc}_{\text{SCM}}$. Then $s(a)$ is an integer.

Let s be a SCMPDS-State, let a be an element of $\text{Data-Loc}_{\text{SCM}}$, and let n be an integer. The functor $\text{Address\_Add}(s,a,n)$ yields an element of $\text{Data-Loc}_{\text{SCM}}$ and is defined by:

(Def. 8) $\text{Address\_Add}(s,a,n) = 2 \cdot |s(a)+n| + 1$.

Let s be a SCMPDS-State and let n be an integer. The functor $\text{jump\_address}(s,n)$ yielding an element of $\text{Instr-Loc}_{\text{SCM}}$ is defined as follows:

(Def. 9) $\text{jump\_address}(s,n) = |((\mathbf{IC}_s\ \mathbf{qua}\ \text{natural number})-2)+2\cdot n|+2$.

Let d be an element of $\text{Data-Loc}_{\text{SCM}}$ and let s be an integer. Then $\langle d,s\rangle$ is a finite sequence of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$.

Let x be an element of SCMPDS-Instr. Let us assume that there exist an element m_1 of $\text{Data-Loc}_{\text{SCM}}$ and I such that $x = \langle I, \langle m_1\rangle\rangle$. The functor $x\,\text{address}_1$ yields an element of $\text{Data-Loc}_{\text{SCM}}$ and is defined as follows:

(Def. 10) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{address}_1 = f_1$.

Next we state the proposition

(33) For every element x of SCMPDS-Instr and for every element m_1 of $\text{Data-Loc}_{\text{SCM}}$ such that $x = \langle I, \langle m_1\rangle\rangle$ holds $x\,\text{address}_1 = m_1$.

Let x be an element of SCMPDS-Instr. Let us assume that there exist an integer r and I such that $x = \langle I, \langle r\rangle\rangle$. The functor $x\,\text{const\_INT}$ yields an integer and is defined as follows:

(Def. 11) There exists a finite sequence f of elements of $\mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{const\_INT} = f_1$.

One can prove the following proposition

(34) For every element x of SCMPDS-Instr and for every integer k such that $x = \langle I, \langle k\rangle\rangle$ holds $x\,\text{const\_INT} = k$.

Let x be an element of SCMPDS-Instr. Let us assume that there exist an element m_1 of $\text{Data-Loc}_{\text{SCM}}$, an integer r, and I such that $x = \langle I, \langle m_1,r\rangle\rangle$. The functor $x\,\text{P21address}$ yields an element of $\text{Data-Loc}_{\text{SCM}}$ and is defined by:

(Def. 12) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P21address} = f_1$.

The functor $x\,\text{P22const}$ yielding an integer is defined as follows:

(Def. 13) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P22const} = f_2$.

We now state the proposition

(35) Let x be an element of SCMPDS-Instr, m_1 be an element of $\text{Data-Loc}_{\text{SCM}}$, and r be an integer. If $x = \langle I, \langle m_1,r\rangle\rangle$, then $x\,\text{P21address} = m_1$ and $x\,\text{P22const} = r$.

Let x be an element of SCMPDS-Instr. Let us assume that there exist an element m_2 of $\text{Data-Loc}_{\text{SCM}}$, integers k_1, k_2, and I such that $x = \langle I, \langle m_2,k_1,k_2\rangle\rangle$. The functor $x\,\text{P31address}$ yielding an element of $\text{Data-Loc}_{\text{SCM}}$ is defined as follows:

(Def. 14) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P31address} = f_1$.

The functor $x\,\text{P32const}$ yielding an integer is defined as follows:

(Def. 15) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P32const} = f_2$.

The functor $x\,\text{P33const}$ yielding an integer is defined by:

(Def. 16) There exists a finite sequence f of elements of $\text{Data-Loc}_{\rm SCM} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P33const} = f_3$.

We now state the proposition

(36) Let x be an element of SCMPDS-Instr, d_1 be an element of $\text{Data-Loc}_{\rm SCM}$, and k_1, k_2 be integers. If $x = \langle\!\langle I, \langle d_1, k_1, k_2 \rangle \rangle\!\rangle$, then $x\,\text{P31address} = d_1$ and $x\,\text{P32const} = k_1$ and $x\,\text{P33const} = k_2$.

Let x be an element of SCMPDS-Instr. Let us assume that there exist elements m_2, m_3 of $\text{Data-Loc}_{\rm SCM}$, integers k_1, k_2, and I such that $x = \langle\!\langle I, \langle m_2, m_3, k_1, k_2 \rangle \rangle\!\rangle$. The functor $x\,\text{P41address}$ yielding an element of $\text{Data-Loc}_{\rm SCM}$ is defined by:

(Def. 17) There exists a finite sequence f of elements of $\text{Data-Loc}_{\rm SCM} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P41address} = f_1$.

The functor $x\,\text{P42address}$ yielding an element of $\text{Data-Loc}_{\rm SCM}$ is defined as follows:

(Def. 18) There exists a finite sequence f of elements of $\text{Data-Loc}_{\rm SCM} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P42address} = f_2$.

The functor $x\,\text{P43const}$ yields an integer and is defined as follows:

(Def. 19) There exists a finite sequence f of elements of $\text{Data-Loc}_{\rm SCM} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P43const} = f_3$.

The functor $x\,\text{P44const}$ yields an integer and is defined by:

(Def. 20) There exists a finite sequence f of elements of $\text{Data-Loc}_{\rm SCM} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P44const} = f_4$.

We now state the proposition

(37) Let x be an element of SCMPDS-Instr, d_1, d_2 be elements of $\text{Data-Loc}_{\rm SCM}$, and k_1, k_2 be integers. If $x = \langle\!\langle I, \langle d_1, d_2, k_1, k_2 \rangle \rangle\!\rangle$, then $x\,\text{P41address} = d_1$ and $x\,\text{P42address} = d_2$ and $x\,\text{P43const} = k_1$ and $x\,\text{P44const} = k_2$.

Let s be a SCMPDS-State and let a be an element of $\text{Data-Loc}_{\rm SCM}$. The functor $\text{PopInstrLoc}(s, a)$ yielding an element of $\text{Instr-Loc}_{\rm SCM}$ is defined as follows:

(Def. 21) $\text{PopInstrLoc}(s, a) = 2 \cdot (|s(a)| \div 2) + 4$.

The natural number RetSP is defined as follows:

(Def. 22) $\text{RetSP} = 0$.

The natural number RetIC is defined by:

(Def. 23) $\text{RetIC} = 1$.

Let x be an element of SCMPDS-Instr and let s be a SCMPDS-State. The functor $\text{Exec-Res}_{\rm SCM}(x, s)$ yielding a SCMPDS-State is defined as follows:

(Def. 24)
$$\text{Exec-Res}_{\rm SCM}(x,s) = \begin{cases}
\text{Chg}_{\rm SCM}(s, \text{jump_address}(s, x\,\text{const_INT})),\ \text{if thereexists} k_1 \text{suchthat} x = \langle\!\langle 0, \langle k_1 \rangle \rangle\!\rangle, \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, x\,\text{P21address}, x\,\text{P22const}), \text{Next}(\mathbf{IC}_s)),\ \text{if thereexist} d_1, k_1 \text{suchthat} x = \langle\!\langle \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\text{P21address}, x\,\text{P22const}), (\mathbf{IC}_s\ \textbf{qua}\ \text{natural number})), \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, x\,\text{address}_1, s(\text{Address_Add}(s, x\,\text{address}_1, \text{RetSP}))), \text{PopInstrLoc}(s, \text{Addr} \\
\text{Chg}_{\rm SCM}(s, (s(\text{Address_Add}(s, x\,\text{P31address}, x\,\text{P32const})) = 0 \to \text{Next}(\mathbf{IC}_s), \text{jump_address}(\\
\text{Chg}_{\rm SCM}(s, (s(\text{Address_Add}(s, x\,\text{P31address}, x\,\text{P32const})) > 0 \to \text{Next}(\mathbf{IC}_s), \text{jump_address}(\\
\text{Chg}_{\rm SCM}(s, (0 > s(\text{Address_Add}(s, x\,\text{P31address}, x\,\text{P32const})) \to \text{Next}(\mathbf{IC}_s), \text{jump_address}(\\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\text{P31address}, x\,\text{P32const}), x\,\text{P33const}), \text{Next}(\mathbf{IC}_s)),\ \text{if} \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\text{P31address}, x\,\text{P32const}), s(\text{Address_Add}(s, x\,\text{P31addr} \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\text{P41address}, x\,\text{P43const}), s(\text{Address_Add}(s, x\,\text{P41addr} \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\text{P41address}, x\,\text{P43const}), s(\text{Address_Add}(s, x\,\text{P41addr} \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\text{P41address}, x\,\text{P43const}), s(\text{Address_Add}(s, x\,\text{P41addr} \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\text{P41address}, x\,\text{P43const}), s(\text{Address_Add}(s, x\,\text{P42addr} \\
\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(\text{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\text{P41address}, x\,\text{P43const}), s(\text{Address_Add}(s, \\
s,\ \text{otherwise.}
\end{cases}$$

Let f be a function from SCMPDS-Instr into $(\prod \text{SCMPDS-OK})^{\Pi \text{SCMPDS-OK}}$ and let x be an element of SCMPDS-Instr. Observe that $f(x)$ is function-like and relation-like.

The function SCMPDS-Exec from SCMPDS-Instr into $(\prod \text{SCMPDS-OK})^{\Pi \text{SCMPDS-OK}}$ is defined as follows:

(Def. 25) For every element x of SCMPDS-Instr and for every SCMPDS-State y holds $(\text{SCMPDS-Exec})(x)(y) = \text{Exec-Res}_{\text{SCM}}(x,y)$.

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