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Some Multi Instructions Defined by Sequence of Instructions of $\mathbf{SCM}_{\mathrm{FSA}}$

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The articles [12], [18], [3], [13], [2], [11], [10], [4], [15], [19], [1], [5], [8], [17], [6], [16], [7], [9], and [14] provide the notation and terminology for this paper.

In this paper m is a natural number.

Let us observe that every finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ is finite.

Let p be a finite sequence and let x, y be sets. Note that $p +\cdot (x,y)$ is finite sequence-like.

We now state four propositions:

(1) For every natural number k holds $|k| = k$.

(2) For all natural numbers a, b, c such that $a \geq c$ and $b \geq c$ and $a -' c = b -' c$ holds $a = b$.

(3) For all natural numbers a, b such that $a \geq b$ holds $a -' b = a - b$.

(4) For all integers a, b such that $a < b$ holds $a \leq b - 1$.

The scheme *CardMono"* deals with a set $\mathcal{A}$, a non empty set $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding a set, and states that:

$\mathcal{A} \approx \{\mathcal{F}(d); d$ ranges over elements of $\mathcal{B} : d \in \mathcal{A}\}$

provided the parameters meet the following requirements:

- $\mathcal{A} \subseteq \mathcal{B}$, and
- For all elements d_1, d_2 of $\mathcal{B}$ such that $d_1 \in \mathcal{A}$ and $d_2 \in \mathcal{A}$ and $\mathcal{F}(d_1) = \mathcal{F}(d_2)$ holds $d_1 = d_2$.

One can prove the following propositions:

(5) For all finite sequences p_1, p_2, q such that $p_1 \subseteq q$ and $p_2 \subseteq q$ and $\mathrm{len}\, p_1 = \mathrm{len}\, p_2$ holds $p_1 = p_2$.

(8)[1] For all finite sequences p, q such that $p \subseteq q$ holds $\mathrm{len}\, p \leq \mathrm{len}\, q$.

(9) For all finite sequences p, q and for every natural number i such that $1 \leq i$ and $i \leq \mathrm{len}\, p$ holds $(p \frown q)(i) = p(i)$.

(10) For all finite sequences p, q and for every natural number i such that $1 \leq i$ and $i \leq \mathrm{len}\, q$ holds $(p \frown q)(\mathrm{len}\, p + i) = q(i)$.

[1] The propositions (6) and (7) have been removed.

(12)[2] For every finite sequence p such that $p \neq \emptyset$ holds $\operatorname{len} p \in \operatorname{dom} p$.

(13) For every set D holds $\operatorname{Flat}(\varepsilon_{D^*}) = \varepsilon_D$.

(14) For every set D and for all finite sequences F, G of elements of D^* holds $\operatorname{Flat}(F \frown G) = \operatorname{Flat}(F) \frown \operatorname{Flat}(G)$.

(15) For every set D and for all elements p, q of D^* holds $\operatorname{Flat}(\langle p, q \rangle) = p \frown q$.

(16) For every set D and for all elements p, q, r of D^* holds $\operatorname{Flat}(\langle p, q, r \rangle) = p \frown q \frown r$.

(17) Let D be a non empty set and p, q be finite sequences of elements of D. If $p \subseteq q$, then there exists a finite sequence p' of elements of D such that $p \frown p' = q$.

(18) Let D be a non empty set, p, q be finite sequences of elements of D, and i be a natural number. If $p \subseteq q$ and $1 \leq i$ and $i \leq \operatorname{len} p$, then $q(i) = p(i)$.

(19) For every set D and for all finite sequences F, G of elements of D^* such that $F \subseteq G$ holds $\operatorname{Flat}(F) \subseteq \operatorname{Flat}(G)$.

(20) For every finite sequence p holds $p \restriction \operatorname{Seg} 0 = \emptyset$.

(21) For all finite sequences f, g holds $f \restriction \operatorname{Seg} 0 = g \restriction \operatorname{Seg} 0$.

(22) For every non empty set D and for every element x of D holds $\langle x \rangle$ is a finite sequence of elements of D.

(23) Let D be a set and p, q be finite sequences of elements of D. Then $p \frown q$ is a finite sequence of elements of D.

Let f be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$. The functor $\operatorname{Load}(f)$ yields a finite partial state of $\mathbf{SCM}_{\rm FSA}$ and is defined as follows:

(Def. 1) $\operatorname{dom} \operatorname{Load}(f) = \{\operatorname{insloc}(m -' 1) : m \in \operatorname{dom} f\}$ and for every natural number k such that $\operatorname{insloc}(k) \in \operatorname{dom} \operatorname{Load}(f)$ holds $(\operatorname{Load}(f))(\operatorname{insloc}(k)) = f_{k+1}$.

We now state several propositions:

(25)[3] For every finite sequence f of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ holds $\operatorname{card} \operatorname{Load}(f) = \operatorname{len} f$.

(26) Let p be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ and k be a natural number. Then $\operatorname{insloc}(k) \in \operatorname{dom} \operatorname{Load}(p)$ if and only if $k+1 \in \operatorname{dom} p$.

(27) For all natural numbers k, n holds $k < n$ iff $0 < k+1$ and $k+1 \leq n$.

(28) For all natural numbers k, n holds $k < n$ iff $1 \leq k+1$ and $k+1 \leq n$.

(29) Let p be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ and k be a natural number. Then $\operatorname{insloc}(k) \in \operatorname{dom} \operatorname{Load}(p)$ if and only if $k < \operatorname{len} p$.

(30) For every non empty finite sequence f of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ holds $1 \in \operatorname{dom} f$ and $\operatorname{insloc}(0) \in \operatorname{dom} \operatorname{Load}(f)$.

(31) For all finite sequences p, q of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ holds $\operatorname{Load}(p) \subseteq \operatorname{Load}(p \frown q)$.

(32) For all finite sequences p, q of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ such that $p \subseteq q$ holds $\operatorname{Load}(p) \subseteq \operatorname{Load}(q)$.

Let a be an integer location and let k be an integer. The functor $a{:=}k$ yields a finite partial state of $\mathbf{SCM}_{\rm FSA}$ and is defined as follows:

[2] The proposition (11) has been removed.
[3] The proposition (24) has been removed.

(Def. 2)(i) There exists a natural number k_1 such that $k_1+1=k$ and $a{:=}k = \mathrm{Load}(\langle a{:=}\mathrm{intloc}(0)\rangle \frown (k_1 \longmapsto \mathrm{AddTo}(a,\mathrm{intloc}(0))) \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$ if $k>0$,

(ii) there exists a natural number k_1 such that $k_1+k=1$ and $a{:=}k = \mathrm{Load}(\langle a{:=}\mathrm{intloc}(0)\rangle \frown (k_1 \longmapsto \mathrm{SubFrom}(a,\mathrm{intloc}(0))) \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$, otherwise.

Let a be an integer location and let k be an integer. The functor $\mathrm{aSeq}(a,k)$ yields a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined by:

(Def. 3)(i) There exists a natural number k_1 such that $k_1+1=k$ and $\mathrm{aSeq}(a,k) = \langle a{:=}\mathrm{intloc}(0)\rangle \frown (k_1 \longmapsto \mathrm{AddTo}(a,\mathrm{intloc}(0)))$ if $k>0$,

(ii) there exists a natural number k_1 such that $k_1+k=1$ and $\mathrm{aSeq}(a,k) = \langle a{:=}\mathrm{intloc}(0)\rangle \frown (k_1 \longmapsto \mathrm{SubFrom}(a,\mathrm{intloc}(0)))$, otherwise.

The following proposition is true

(33) For every integer location a and for every integer k holds $a{:=}k = \mathrm{Load}((\mathrm{aSeq}(a,k)) \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$.

Let f be a finite sequence location and let p be a finite sequence of elements of $\mathbb{Z}$. The functor $\mathrm{aSeq}(f,p)$ yields a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined by the condition (Def. 4).

(Def. 4) There exists a finite sequence p_3 of elements of (the instructions of $\mathbf{SCM}_{\mathrm{FSA}})^*$ such that

(i) $\mathrm{len}\, p_3 = \mathrm{len}\, p$,

(ii) for every natural number k such that $1 \leq k$ and $k \leq \mathrm{len}\, p$ there exists an integer i such that $i = p(k)$ and $p_3(k) = (\mathrm{aSeq}(\mathrm{intloc}(1),k)) \frown \mathrm{aSeq}(\mathrm{intloc}(2),i) \frown \langle f_{\mathrm{intloc}(1)}{:=}\mathrm{intloc}(2)\rangle$, and

(iii) $\mathrm{aSeq}(f,p) = \mathrm{Flat}(p_3)$.

Let f be a finite sequence location and let p be a finite sequence of elements of $\mathbb{Z}$. The functor $f{:=}p$ yielding a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ is defined by:

(Def. 5) $f{:=}p = \mathrm{Load}((\mathrm{aSeq}(\mathrm{intloc}(1),\mathrm{len}\, p)) \frown \langle f{:=}\langle \underbrace{0,\ldots,0}_{\mathrm{intloc}(1)} \rangle\rangle \frown \mathrm{aSeq}(f,p) \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$.

We now state several propositions:

(34) For every integer location a holds $a{:=}1 = \mathrm{Load}(\langle a{:=}\mathrm{intloc}(0)\rangle \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$.

(35) For every integer location a holds $a{:=}0 = \mathrm{Load}(\langle a{:=}\mathrm{intloc}(0)\rangle \frown \langle \mathrm{SubFrom}(a,\mathrm{intloc}(0))\rangle \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$.

(36) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $s(\mathrm{intloc}(0)) = 1$. Let c_0 be a natural number. Suppose $\mathbf{IC}_s = \mathrm{insloc}(c_0)$. Let a be an integer location and k be an integer. Suppose $a \neq \mathrm{intloc}(0)$ and for every natural number c such that $c \in \mathrm{dom}\,\mathrm{aSeq}(a,k)$ holds $(\mathrm{aSeq}(a,k))(c) = s(\mathrm{insloc}((c_0+c) -' 1))$. Then

(i) for every natural number i such that $i \leq \mathrm{len}\,\mathrm{aSeq}(a,k)$ holds $\mathbf{IC}_{(\mathrm{Computation}(s))(i)} = \mathrm{insloc}(c_0+i)$ and for every integer location b such that $b \neq a$ holds $(\mathrm{Computation}(s))(i)(b) = s(b)$ and for every finite sequence location f holds $(\mathrm{Computation}(s))(i)(f) = s(f)$, and

(ii) $(\mathrm{Computation}(s))(\mathrm{len}\,\mathrm{aSeq}(a,k))(a) = k$.

(37) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s = \mathrm{insloc}(0)$ and $s(\mathrm{intloc}(0)) = 1$. Let a be an integer location and k be an integer. Suppose $\mathrm{Load}(\mathrm{aSeq}(a,k)) \subseteq s$ and $a \neq \mathrm{intloc}(0)$. Then

(i) for every natural number i such that $i \leq \mathrm{len}\,\mathrm{aSeq}(a,k)$ holds $\mathbf{IC}_{(\mathrm{Computation}(s))(i)} = \mathrm{insloc}(i)$ and for every integer location b such that $b \neq a$ holds $(\mathrm{Computation}(s))(i)(b) = s(b)$ and for every finite sequence location f holds $(\mathrm{Computation}(s))(i)(f) = s(f)$, and

(ii) $(\mathrm{Computation}(s))(\mathrm{len}\,\mathrm{aSeq}(a,k))(a) = k$.

(38) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s = \mathrm{insloc}(0)$ and $s(\mathrm{intloc}(0)) = 1$. Let a be an integer location and k be an integer. Suppose $a{:=}k \subseteq s$ and $a \neq \mathrm{intloc}(0)$. Then

(i) s is halting,

(ii) $(\mathrm{Result}(s))(a) = k$,

(iii) for every integer location b such that $b \neq a$ holds $(\mathrm{Result}(s))(b) = s(b)$, and

(iv) for every finite sequence location f holds $(\mathrm{Result}(s))(f) = s(f)$.

(39) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s = \mathrm{insloc}(0)$ and $s(\mathrm{intloc}(0)) = 1$. Let f be a finite sequence location and p be a finite sequence of elements of $\mathbb{Z}$. Suppose $f{:=}p \subseteq s$. Then

(i) s is halting,

(ii) $(\mathrm{Result}(s))(f) = p$,

(iii) for every integer location b such that $b \neq \mathrm{intloc}(1)$ and $b \neq \mathrm{intloc}(2)$ holds $(\mathrm{Result}(s))(b) = s(b)$, and

(iv) for every finite sequence location g such that $g \neq f$ holds $(\mathrm{Result}(s))(g) = s(g)$.

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