

Relocability for $\mathbf{SCM}_{\text{FSA}}$

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MML Identifier: SCMFSA_5.

WWW: http://mizar.org/JFM/Vol8/scmfsa_5.html

The articles [13], [18], [11], [1], [17], [2], [6], [19], [4], [3], [7], [9], [5], [8], [10], [14], [12], [16], and [15] provide the notation and terminology for this paper.

1. RELOCABILITY

In this paper j, k denote natural numbers.

Let p be a finite partial state of $\mathbf{SCM}_{\text{FSA}}$ and let k be a natural number. The functor $\text{Relocated}(p, k)$ yields a finite partial state of $\mathbf{SCM}_{\text{FSA}}$ and is defined by:

(Def. 1) $\text{Relocated}(p, k) = \text{Start-At}(\mathbf{IC}_p + k) + \cdot \text{IncAddr}(\text{Shift}(\text{ProgramPart}(p), k), k) + \cdot \text{DataPart}(p)$.

The following propositions are true:

- (1) For every finite partial state p of $\mathbf{SCM}_{\text{FSA}}$ and for every natural number k holds $\text{DataPart}(\text{Relocated}(p, k)) = \text{DataPart}(p)$.
- (2) For every finite partial state p of $\mathbf{SCM}_{\text{FSA}}$ and for every natural number k holds $\text{ProgramPart}(\text{Relocated}(p, k)) = \text{IncAddr}(\text{Shift}(\text{ProgramPart}(p), k), k)$.
- (3) For every finite partial state p of $\mathbf{SCM}_{\text{FSA}}$ holds $\text{dom ProgramPart}(\text{Relocated}(p, k)) = \{\text{insloc}(j+k) : \text{insloc}(j) \in \text{dom ProgramPart}(p)\}$.
- (4) Let p be a finite partial state of $\mathbf{SCM}_{\text{FSA}}$, k be a natural number, and l be an instruction-location of $\mathbf{SCM}_{\text{FSA}}$. Then $l \in \text{dom } p$ if and only if $l+k \in \text{dom Relocated}(p, k)$.
- (5) For every finite partial state p of $\mathbf{SCM}_{\text{FSA}}$ and for every natural number k holds $\mathbf{IC}_{\text{SCM}_{\text{FSA}}} \in \text{dom Relocated}(p, k)$.
- (6) For every finite partial state p of $\mathbf{SCM}_{\text{FSA}}$ and for every natural number k holds $\mathbf{IC}_{\text{Relocated}(p, k)} = \mathbf{IC}_p + k$.
- (7) Let p be a finite partial state of $\mathbf{SCM}_{\text{FSA}}$, k be a natural number, l_1 be an instruction-location of $\mathbf{SCM}_{\text{FSA}}$, and I be an instruction of $\mathbf{SCM}_{\text{FSA}}$. If $l_1 \in \text{dom ProgramPart}(p)$ and $I = p(l_1)$, then $\text{IncAddr}(I, k) = (\text{Relocated}(p, k))(l_1 + k)$.
- (8) For every finite partial state p of $\mathbf{SCM}_{\text{FSA}}$ and for every natural number k holds $\text{Start-At}(\mathbf{IC}_p + k) \subseteq \text{Relocated}(p, k)$.
- (9) Let s be a data-only finite partial state of $\mathbf{SCM}_{\text{FSA}}$, p be a finite partial state of $\mathbf{SCM}_{\text{FSA}}$, and k be a natural number. If $\mathbf{IC}_{\text{SCM}_{\text{FSA}}} \in \text{dom } p$, then $\text{Relocated}(p + \cdot s, k) = \text{Relocated}(p, k) + \cdot s$.

- (10) Let k be a natural number, p be an autonomic finite partial state of $\mathbf{SCM}_{\text{FSA}}$, and s_1, s_2 be states of $\mathbf{SCM}_{\text{FSA}}$. If $p \subseteq s_1$ and $\text{Relocated}(p, k) \subseteq s_2$, then $p \subseteq s_1 + \cdot s_2 \upharpoonright (\text{Int-Locations} \cup \text{FinSeq-Locations})$.

2. MAIN THEOREMS OF RELOCABILITY

One can prove the following propositions:

- (11) Let k be a natural number and p be an autonomic finite partial state of $\mathbf{SCM}_{\text{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \in \text{dom } p$. Let s be a state of $\mathbf{SCM}_{\text{FSA}}$. Suppose $p \subseteq s$. Let i be a natural number. Then $(\text{Computation}(s + \cdot \text{Relocated}(p, k)))(i) = (\text{Computation}(s))(i) + \cdot \text{Start-At}(\mathbf{IC}_{(\text{Computation}(s))(i)} + k) + \cdot \text{ProgramPart}(\text{Relocated}(p, k))$.
- (12) Let k be a natural number, p be an autonomic finite partial state of $\mathbf{SCM}_{\text{FSA}}$, and s_1, s_2, s_3 be states of $\mathbf{SCM}_{\text{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \in \text{dom } p$ and $p \subseteq s_1$ and $\text{Relocated}(p, k) \subseteq s_2$ and $s_3 = s_1 + \cdot s_2 \upharpoonright (\text{Int-Locations} \cup \text{FinSeq-Locations})$. Let i be a natural number. Then $\mathbf{IC}_{(\text{Computation}(s_1))(i)} + k = \mathbf{IC}_{(\text{Computation}(s_2))(i)}$ and $\text{IncAddr}(\text{CurInstr}((\text{Computation}(s_1))(i)), k) = \text{CurInstr}((\text{Computation}(s_2))(i))$ and $(\text{Computation}(s_1))(i) \upharpoonright \text{dom DataPart}(\text{Computation}(s_2))(i) \upharpoonright \text{dom DataPart}(\text{Relocated}(p, k))$ and $(\text{Computation}(s_3))(i) \upharpoonright (\text{Int-Locations} \cup \text{FinSeq-Locations}) = (\text{Computation}(s_2))(i) \upharpoonright (\text{Int-Locations} \cup \text{FinSeq-Locations})$.
- (13) Let p be an autonomic finite partial state of $\mathbf{SCM}_{\text{FSA}}$ and k be a natural number. If $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \in \text{dom } p$, then p is halting iff $\text{Relocated}(p, k)$ is halting.
- (14) Let k be a natural number and p be an autonomic finite partial state of $\mathbf{SCM}_{\text{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \in \text{dom } p$. Let s be a state of $\mathbf{SCM}_{\text{FSA}}$. Suppose $\text{Relocated}(p, k) \subseteq s$. Let i be a natural number. Then $(\text{Computation}(s))(i) = (\text{Computation}(s + \cdot p))(i) + \cdot \text{Start-At}(\mathbf{IC}_{(\text{Computation}(s + \cdot p))(i)} + k) + \cdot s \upharpoonright \text{dom ProgramPart}(p) + \cdot \text{ProgramPart}(\text{Relocated}(p, k))$.
- (15) Let k be a natural number and p be a finite partial state of $\mathbf{SCM}_{\text{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \in \text{dom } p$. Let s be a state of $\mathbf{SCM}_{\text{FSA}}$. Suppose $p \subseteq s$ and $\text{Relocated}(p, k)$ is autonomic. Let i be a natural number. Then $(\text{Computation}(s))(i) = (\text{Computation}(s + \cdot \text{Relocated}(p, k)))(i) + \cdot \text{Start-At}(\mathbf{IC}_{(\text{Computation}(s + \cdot \text{Relocated}(p, k)))(i)} - k) + \cdot s \upharpoonright \text{dom ProgramPart}(\text{Relocated}(p, k))$.
- (16) Let p be a finite partial state of $\mathbf{SCM}_{\text{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \in \text{dom } p$. Let k be a natural number. Then p is autonomic if and only if $\text{Relocated}(p, k)$ is autonomic.
- (17) Let p be a halting autonomic finite partial state of $\mathbf{SCM}_{\text{FSA}}$. If $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \in \text{dom } p$, then for every natural number k holds $\text{DataPart}(\text{Result}(p)) = \text{DataPart}(\text{Result}(\text{Relocated}(p, k)))$.
- (18) Let F be a partial function from $\text{FinPartSt}(\mathbf{SCM}_{\text{FSA}})$ to $\text{FinPartSt}(\mathbf{SCM}_{\text{FSA}})$ and p be a finite partial state of $\mathbf{SCM}_{\text{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \in \text{dom } p$ and F is data-only. Let k be a natural number. Then p computes F if and only if $\text{Relocated}(p, k)$ computes F .

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Received February 22, 1996

Published January 2, 2004
