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The `while` Macro Instructions of $\mathbf{SCM}_{\rm FSA}$. Part II

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Summary. An attempt to use the `while` macro, [16], was the origin of writing this article. The `while` semantics, as given by J.-C. Chen, is slightly extended by weakening its correctness conditions and this forced a quite straightforward remake of a number of theorems from [16]. Numerous additional properties of the `while` macro are then proven. In the last section, we define a macro instruction computing the `fusc` function (see the SCM program computing the same function in [12]) and prove its correctness.

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The articles [23], [32], [24], [25], [22], [7], [9], [30], [8], [20], [33], [13], [14], [15], [11], [17], [26], [10], [21], [29], [27], [28], [5], [19], [6], [4], [3], [1], [2], [16], [18], and [31] provide the notation and terminology for this paper.

1. Arithmetic Preliminaries

We adopt the following convention: k, m, n are natural numbers, i, j are integers, and r is a real number.

The scheme *MinPred* deals with a unary functor $\mathcal{F}$ yielding a natural number and a unary predicate $\mathcal{P}$, and states that:

There exists k such that $\mathcal{P}[k]$ and for every n such that $\mathcal{P}[n]$ holds $k \leq n$

provided the parameters meet the following condition:

- For every k holds $\mathcal{F}(k+1) < \mathcal{F}(k)$ or $\mathcal{P}[k]$.

One can prove the following propositions:

(1) n is odd iff there exists a natural number k such that $n = 2 \cdot k + 1$.

(2) For every integer i such that $i \leq r$ holds $i \leq \lfloor r \rfloor$.

(3) If $0 < n$, then $0 \leq (m$ **qua** integer$) \div n$.

(4) If $0 < i$ and $1 < j$, then $i \div j < i$.

(5) $(m$ **qua** integer$) \div n = m \div n$ and $(m$ **qua** integer$) \bmod n = m \bmod n$.

2. $\mathbf{SCM}_{\rm FSA}$ Preliminaries

In the sequel l denotes an instruction-location of $\mathbf{SCM}_{\rm FSA}$ and i denotes an instruction of $\mathbf{SCM}_{\rm FSA}$.

We now state several propositions:

(6) Let N be a non empty set with non empty elements, S be a halting IC-Ins-separated definite non empty non void AMI over N, s be a state of S, and k be a natural number. If $\mathrm{CurInstr}((\mathrm{Computation}(s))(k)) = \mathbf{halt}_S$, then $(\mathrm{Computation}(s))(\mathrm{LifeSpan}(s)) = (\mathrm{Computation}(s))(k)$.

(7) $\mathrm{UsedIntLoc}(l \dot{\longmapsto} i) = \mathrm{UsedIntLoc}(i)$.

(8) $\mathrm{UsedInt}^* \mathrm{Loc}(l \dot{\longmapsto} i) = \mathrm{UsedInt}^* \mathrm{Loc}(i)$.

(9) $\mathrm{UsedIntLoc}(\mathrm{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}) = \emptyset$.

(10) $\mathrm{UsedInt}^* \mathrm{Loc}(\mathrm{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}) = \emptyset$.

(11) $\mathrm{UsedIntLoc}(\mathrm{Goto}(l)) = \emptyset$.

(12) $\mathrm{UsedInt}^* \mathrm{Loc}(\mathrm{Goto}(l)) = \emptyset$.

For simplicity, we follow the rules: s, s_1, s_2 denote states of $\mathbf{SCM}_{\mathrm{FSA}}$, a denotes a read-write integer location, b denotes an integer location, I, J denote macro instructions, I_1 denotes a good macro instruction, and i, j, k denote natural numbers.

One can prove the following four propositions:

(13) $\mathrm{UsedIntLoc}(\mathbf{if}\ b = 0\ \mathbf{then}\ I\ \mathbf{else}\ J) = \{b\} \cup \mathrm{UsedIntLoc}(I) \cup \mathrm{UsedIntLoc}(J)$.

(14) For every integer location a holds $\mathrm{UsedInt}^* \mathrm{Loc}(\mathbf{if}\ a = 0\ \mathbf{then}\ I\ \mathbf{else}\ J) = \mathrm{UsedInt}^* \mathrm{Loc}(I) \cup \mathrm{UsedInt}^* \mathrm{Loc}(J)$.

(15) $\mathrm{UsedIntLoc}(\mathbf{if}\ b > 0\ \mathbf{then}\ I\ \mathbf{else}\ J) = \{b\} \cup \mathrm{UsedIntLoc}(I) \cup \mathrm{UsedIntLoc}(J)$.

(16) $\mathrm{UsedInt}^* \mathrm{Loc}(\mathbf{if}\ b > 0\ \mathbf{then}\ I\ \mathbf{else}\ J) = \mathrm{UsedInt}^* \mathrm{Loc}(I) \cup \mathrm{UsedInt}^* \mathrm{Loc}(J)$.

3. THE while=0 MACRO INSTRUCTION

The following two propositions are true:

(17) $\mathrm{UsedIntLoc}(\mathbf{while}\ b = 0\ \mathbf{do}\ I) = \{b\} \cup \mathrm{UsedIntLoc}(I)$.

(18) $\mathrm{UsedInt}^* \mathrm{Loc}(\mathbf{while}\ b = 0\ \mathbf{do}\ I) = \mathrm{UsedInt}^* \mathrm{Loc}(I)$.

Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, let a be a read-write integer location, and let I be a macro instruction. The predicate ProperBodyWhile=0(a, I, s) is defined as follows:

(Def. 1) For every natural number k such that $(\mathit{StepWhile{=}0}(a,I,s))(k)(a) = 0$ holds I is closed on $(\mathit{StepWhile{=}0}(a,I,s))(k)$ and halting on $(\mathit{StepWhile{=}0}(a,I,s))(k)$.

The predicate WithVariantWhile=0(a, I, s) is defined by the condition (Def. 2).

(Def. 2) There exists a function f from $\prod$(the object kind of $\mathbf{SCM}_{\mathrm{FSA}}$) into $\mathbb{N}$ such that for every natural number k holds

$f((\mathit{StepWhile{=}0}(a,I,s))(k+1)) < f((\mathit{StepWhile{=}0}(a,I,s))(k))$ or $(\mathit{StepWhile{=}0}(a,I,s))(k)(a) \neq 0$.

We now state several propositions:

(19) For every parahalting macro instruction I holds ProperBodyWhile=0(a, I, s).

(20) If ProperBodyWhile=0(a, I, s) and WithVariantWhile=0(a, I, s), then $\mathbf{while}\ a = 0\ \mathbf{do}\ I$ is halting on s and $\mathbf{while}\ a = 0\ \mathbf{do}\ I$ is closed on s.

(21) For every parahalting macro instruction I such that WithVariantWhile=0(a, I, s) holds $\mathbf{while}\ a = 0\ \mathbf{do}\ I$ is halting on s and $\mathbf{while}\ a = 0\ \mathbf{do}\ I$ is closed on s.

(22) If (**while** $a = 0$ **do** $I) +\cdot S_1 \subseteq s$ and $s(a) \neq 0$, then $\mathrm{LifeSpan}(s) = 4$ and for every natural number k holds $(\mathrm{Computation}(s))(k) \restriction D = s \restriction D$, where $S_1 = \mathrm{Start\text{-}At}(\mathrm{insloc}(0))$ and $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

(23) If I is closed on s and halting on s and $s(a) = 0$, then $(\mathrm{Computation}(s +\cdot ((\mathbf{while}\ a = 0\ \mathbf{do}\ I) +\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(\mathrm{LifeSpan}(s +\cdot (I +\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))) + 3) \restriction D = (\mathrm{Computation}(s +\cdot (I +\cdot \mathrm{Start\text{-}At}(\mathrm{ins}$ where $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

(24) If $(\mathit{StepWhile{=}0}(a, I, s))(k)(a) \neq 0$, then $(\mathit{StepWhile{=}0}(a, I, s))(k+1) \restriction D = (\mathit{StepWhile{=}0}(a, I, s))(k) \restriction D$, where $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

(25) Suppose I is halting on $\mathrm{Initialize}((\mathit{StepWhile{=}0}(a, I, s))(k))$, closed on $\mathrm{Initialize}((\mathit{StepWhile{=}0}(a, I, s))(k))$, and parahalting and $(\mathit{StepWhile{=}0}(a, I, s))(k)(a) = 0$ and $(\mathit{StepWhile{=}0}(a, I, s))(k)(\mathrm{intloc}(0)) = 1$. Then $(\mathit{StepWhile{=}0}(a, I, s))(k+1) \restriction D = \mathrm{IExec}(I, (\mathit{StepWhile{=}0}(a, I, s))(k)) \restriction D$, where $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

(26) If $\mathrm{ProperBodyWhile{=}0}(a, I_1, s)$ or I_1 is parahalting and if $s(\mathrm{intloc}(0)) = 1$, then for every k holds $(\mathit{StepWhile{=}0}(a, I_1, s))(k)(\mathrm{intloc}(0)) = 1$.

(27) If $\mathrm{ProperBodyWhile{=}0}(a, I, s_1)$ and $s_1 \restriction D = s_2 \restriction D$, then for every k holds $(\mathit{StepWhile{=}0}(a, I, s_1))(k) \restriction D = (\mathit{StepWhile{=}0}(a, I, s_2))(k) \restriction D$, where $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, let a be a read-write integer location, and let I be a macro instruction. Let us assume that $\mathrm{ProperBodyWhile{=}0}(a, I, s)$ or I is parahalting and $\mathrm{WithVariantWhile{=}0}(a, I, s)$. The functor $\mathit{ExitsAtWhile{=}0}(a, I, s)$ yields a natural number and is defined by the condition (Def. 3).

(Def. 3) There exists a natural number k such that

(i) $\mathit{ExitsAtWhile{=}0}(a, I, s) = k$,

(ii) $(\mathit{StepWhile{=}0}(a, I, s))(k)(a) \neq 0$,

(iii) for every natural number i such that $(\mathit{StepWhile{=}0}(a, I, s))(i)(a) \neq 0$ holds $k \leq i$, and

(iv) $(\mathrm{Computation}(s +\cdot ((\mathbf{while}\ a = 0\ \mathbf{do}\ I) +\cdot S_1)))(\mathrm{LifeSpan}(s +\cdot ((\mathbf{while}\ a = 0\ \mathbf{do}\ I) +\cdot S_1))) \restriction D = (\mathit{StepWhile{=}0}(a, I, s))(k) \restriction D$,

where $S_1 = \mathrm{Start\text{-}At}(\mathrm{insloc}(0))$ and $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

Next we state two propositions:

(28) If $s(\mathrm{intloc}(0)) = 1$ and $s(a) \neq 0$, then $\mathrm{IExec}(\mathbf{while}\ a = 0\ \mathbf{do}\ I, s) \restriction D = s \restriction D$, where $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

(29) If $\mathrm{ProperBodyWhile{=}0}(a, I, \mathrm{Initialize}(s))$ or I is parahalting and if $\mathrm{WithVariantWhile{=}0}(a, I, \mathrm{Initialize}(s))$, then $\mathrm{IExec}(\mathbf{while}\ a = 0\ \mathbf{do}\ I, s) \restriction D = (\mathit{StepWhile{=}0}(a, I, \mathrm{Initialize}(s)))(\mathit{ExitsAtWhile{=}0}(a, I, \mathrm{Initialize}(s))) \restriction D$, where $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

4. THE while>0 MACRO INSTRUCTION

Next we state two propositions:

(30) $\mathrm{UsedIntLoc}(\mathbf{while}\ b > 0\ \mathbf{do}\ I) = \{b\} \cup \mathrm{UsedIntLoc}(I)$.

(31) $\mathrm{UsedInt}^{*}\mathrm{Loc}(\mathbf{while}\ b > 0\ \mathbf{do}\ I) = \mathrm{UsedInt}^{*}\mathrm{Loc}(I)$.

Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, let a be a read-write integer location, and let I be a macro instruction. The predicate $\mathrm{ProperBodyWhile{>}0}(a, I, s)$ is defined as follows:

(Def. 4) For every natural number k such that $(\mathit{StepWhile{>}0}(a, I, s))(k)(a) > 0$ holds I is closed on $(\mathit{StepWhile{>}0}(a, I, s))(k)$ and halting on $(\mathit{StepWhile{>}0}(a, I, s))(k)$.

The predicate $\mathrm{WithVariantWhile{>}0}(a, I, s)$ is defined by the condition (Def. 5).

(Def. 5) There exists a function f from $\prod$(the object kind of $\mathbf{SCM}_{\mathrm{FSA}}$) into $\mathbb{N}$ such that for every natural number k holds

$f((\mathit{StepWhile}{>}0(a,I,s))(k+1)) < f((\mathit{StepWhile}{>}0(a,I,s))(k))$ or $(\mathit{StepWhile}{>}0(a,I,s))(k)(a) \leq 0$.

We now state several propositions:

(32) For every parahalting macro instruction I holds ProperBodyWhile>0(a,I,s).

(33) If ProperBodyWhile>0(a,I,s) and WithVariantWhile>0(a,I,s), then **while** $a>0$ **do** I is halting on s and **while** $a>0$ **do** I is closed on s.

(34) For every parahalting macro instruction I such that WithVariantWhile>0(a,I,s) holds **while** $a>0$ **do** I is halting on s and **while** $a>0$ **do** I is closed on s.

(35) If $(\textbf{while } a>0 \textbf{ do } I) {+\cdot} S_1 \subseteq s$ and $s(a) \leq 0$, then $\mathrm{LifeSpan}(s) = 4$ and for every natural number k holds $(\mathrm{Computation}(s))(k) \restriction D = s \restriction D$, where $S_1 = \text{Start-At}(\mathrm{insloc}(0))$ and $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(36) If I is closed on s and halting on s and $s(a) > 0$, then $(\mathrm{Computation}(s {+\cdot} ((\textbf{while } a > 0 \textbf{ do } I) {+\cdot} \text{Start-At}(\mathrm{insloc}(0)))))(\mathrm{LifeSpan}(s {+\cdot} (I {+\cdot} \text{Start-At}(\mathrm{insloc}(0)))) + 3) \restriction D = (\mathrm{Computation}(s {+\cdot} (I {+\cdot}$ Start-At(ins
where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(37) If $(\mathit{StepWhile}{>}0(a,I,s))(k)(a) \leq 0$, then $(\mathit{StepWhile}{>}0(a,I,s))(k+1) \restriction D = (\mathit{StepWhile}{>}0(a,I,s))(k) \restriction D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(38) Suppose I is halting on $\mathrm{Initialize}((\mathit{StepWhile}{>}0(a,I,s))(k))$, closed on $\mathrm{Initialize}((\mathit{StepWhile}{>}0(a,I,s))(k))$, and parahalting and $(\mathit{StepWhile}{>}0(a,I,s))(k)(a) > 0$ and $(\mathit{StepWhile}{>}0(a,I,s))(k)(\mathrm{intloc}(0)) = 1$. Then $(\mathit{StepWhile}{>}0(a,I,s))(k+1) \restriction D = \mathrm{IExec}(I, (\mathit{StepWhile}{>}0(a,I,s))(k)) \restriction D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(39) If ProperBodyWhile>0(a,I_1,s) or I_1 is parahalting and if $s(\mathrm{intloc}(0)) = 1$, then for every k holds $(\mathit{StepWhile}{>}0(a,I_1,s))(k)(\mathrm{intloc}(0)) = 1$.

(40) If ProperBodyWhile>0(a,I,s_1) and $s_1 \restriction D = s_2 \restriction D$, then for every k holds $(\mathit{StepWhile}{>}0(a,I,s_1))(k) \restriction D = (\mathit{StepWhile}{>}0(a,I,s_2))(k) \restriction D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, let a be a read-write integer location, and let I be a macro instruction. Let us assume that ProperBodyWhile>0(a,I,s) or I is parahalting and WithVariantWhile>0(a,I,s). The functor $\mathit{ExitsAtWhile}{>}0(a,I,s)$ yielding a natural number is defined by the condition (Def. 6).

(Def. 6) There exists a natural number k such that

(i) $\mathit{ExitsAtWhile}{>}0(a,I,s) = k$,

(ii) $(\mathit{StepWhile}{>}0(a,I,s))(k)(a) \leq 0$,

(iii) for every natural number i such that $(\mathit{StepWhile}{>}0(a,I,s))(i)(a) \leq 0$ holds $k \leq i$, and

(iv) $(\mathrm{Computation}(s {+\cdot} ((\textbf{while } a>0 \textbf{ do } I) {+\cdot} S_1)))(\mathrm{LifeSpan}(s {+\cdot} ((\textbf{while } a>0 \textbf{ do } I) {+\cdot} S_1))) \restriction D = (\mathit{StepWhile}{>}0(a,I,s))(k) \restriction D$,

where $S_1 = \text{Start-At}(\mathrm{insloc}(0))$ and $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

Next we state several propositions:

(41) If $s(\mathrm{intloc}(0)) = 1$ and $s(a) \leq 0$, then $\mathrm{IExec}(\textbf{while } a>0 \textbf{ do } I, s) \restriction D = s \restriction D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(42) If ProperBodyWhile>0$(a,I,\mathrm{Initialize}(s))$ or I is parahalting and if WithVariantWhile>0$(a,I,\mathrm{Initialize}(s))$, then $\mathrm{IExec}(\textbf{while } a>0 \textbf{ do } I, s) \restriction D = (\mathit{StepWhile}{>}0(a,I,\mathrm{Initialize}(s)))(\mathit{ExitsAtWhile}{>}0(a,I,\mathrm{Initialize}(s))) \restriction D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(43) If $(StepWhile{>}0(a,I,s))(k)(a) \leq 0$, then for every natural number n such that $k \leq n$ holds $(StepWhile{>}0(a,I,s))(n) \restriction D = (StepWhile{>}0(a,I,s))(k) \restriction D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(44) If $s_1 \restriction D = s_2 \restriction D$ and ProperBodyWhile$>0(a,I,s_1)$, then ProperBodyWhile$>0(a,I,s_2)$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(45) Suppose $s(\text{intloc}(0)) = 1$ and ProperBodyWhile$>0(a,I_1,s)$ and WithVariantWhile$>0(a,I_1,s)$. Let given i, j. Suppose $i \neq j$ and $i \leq ExitsAtWhile{>}0(a,I_1,s)$ and $j \leq ExitsAtWhile{>}0(a,I_1,s)$. Then $(StepWhile{>}0(a,I_1,s))(i) \neq (StepWhile{>}0(a,I_1,s))(j)$ and $(StepWhile{>}0(a,I_1,s))(i) \restriction D \neq (StepWhile{>}0(a,I_1,s))(j) \restriction D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

Let f be a function from $\prod$(the object kind of $\mathbf{SCM}_{\rm FSA}$) into $\mathbb{N}$. We say that f is on data only if and only if:

(Def. 7) For all s_1, s_2 such that $s_1 \restriction D = s_2 \restriction D$ holds $f(s_1) = f(s_2)$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

We now state two propositions:

(46) Suppose $s(\text{intloc}(0)) = 1$ and ProperBodyWhile$>0(a,I_1,s)$ and WithVariantWhile$>0(a,I_1,s)$. Then there exists a function f from $\prod$(the object kind of $\mathbf{SCM}_{\rm FSA}$) into $\mathbb{N}$ such that f is on data only and for every natural number k holds $f((StepWhile{>}0(a,I_1,s))(k+1)) < f((StepWhile{>}0(a,I_1,s))(k))$ or $(StepWhile{>}0(a,I_1,s))(k)(a) \leq 0$.

(47) If $s_1(\text{intloc}(0)) = 1$ and $s_1 \restriction D = s_2 \restriction D$ and ProperBodyWhile$>0(a,I_1,s_1)$ and WithVariantWhile$>0(a,I_1,s_1)$, then WithVariantWhile$>0(a,I_1,s_2)$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

5. A Macro for the fusc Function

Let N, r_1 be integer locations. The functor Fusc_macro(N,r_1) yielding a macro instruction is defined by:

(Def. 8) Fusc_macro(N,r_1) = SubFrom(r_1,r_1); $(n_1{:=}\text{intloc}(0))$; $(a_1{:=}N)$; (**while** $a_1 > 0$ **do** $((r_2{:=}2)$; Divide(a_1,r_2); (**if** $r_2 = 0$ **then** Macro(AddTo(n_1,r_1)) **else** Macro(AddTo(r_1,n_1)))))), where $n_1 = 1^{\rm st}$-RWNotIn$(\{N,r_1\})$, $a_1 = 2^{\rm nd}$-RWNotIn$(\{N,r_1\})$, and $r_2 = 3^{\rm rd}$-RWNotIn$(\{N,r_1\})$.

One can prove the following proposition

(48) Let N, r_1 be read-write integer locations. Suppose $N \neq r_1$. Let n be a natural number. If $n = s(N)$, then $(\text{IExec}(\text{Fusc\_macro}(N,r_1),s))(r_1) = \text{Fusc}(n)$ and $(\text{IExec}(\text{Fusc\_macro}(N,r_1),s))(N) = n$.

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