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Conditional Branch Macro Instructions of $\mathrm{SCM}_{\mathrm{FSA}}$. Part II

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The articles [15], [6], [21], [23], [9], [11], [8], [5], [7], [10], [12], [16], [14], [20], [17], [18], [19], [13], [3], [4], [2], [1], and [22] provide the notation and terminology for this paper.

The following propositions are true:

(1) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\, s$.

(2) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every instruction-location l of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $l \in \mathrm{dom}\, s$.

(3) For every macro instruction I and for every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ such that I is closed on s holds $\mathrm{insloc}(0) \in \mathrm{dom}\, I$.

(4) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ and for all instruction-locations l_1, l_2 of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $s+\cdot \text{Start-At}(l_1)+\cdot \text{Start-At}(l_2) = s+\cdot \text{Start-At}(l_2)$.

(5) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every macro instruction I holds $\text{Initialize}(s)\restriction(\text{Int-Locations} \cup \text{FinSeq-Locations}) = (s+\cdot \text{Initialized}(I))\restriction(\text{Int-Locations} \cup \text{FinSeq-Locations})$.

(6) Let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. If $s_1\restriction(\text{Int-Locations} \cup \text{FinSeq-Locations}) = s_2\restriction(\text{Int-Locations} \cup \text{FinSeq-Locations})$, then if I is closed on s_1, then I is closed on s_2.

(7) Let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$ and I, J be macro instructions. Suppose $s_1\restriction(\text{Int-Locations} \cup \text{FinSeq-Locations}) = s_2\restriction(\text{Int-Locations} \cup \text{FinSeq-Locations})$. Then $s_1+\cdot(I+\cdot \text{Start-At}(\text{insloc}(0)))$ and $s_2+\cdot(J+\cdot \text{Start-At}(\text{insloc}(0)))$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

(8) Let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. Suppose $s_1\restriction(\text{Int-Locations} \cup \text{FinSeq-Locations}) = s_2\restriction(\text{Int-Locations} \cup \text{FinSeq-Locations})$. Suppose I is closed on s_1 and halting on s_1. Then I is closed on s_2 and halting on s_2.

(9) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ and for all macro instructions I, J holds I is closed on $\text{Initialize}(s)$ iff I is closed on $s+\cdot \text{Initialized}(J)$.

(10) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. Then I is closed on s if and only if I is closed on $s+\cdot(I+\cdot \text{Start-At}(l))$.

(11) Let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. Suppose $I+\cdot\,\text{Start-At}(\text{insloc}(0)) \subseteq s_1$ and I is closed on s_1. Let n be a natural number. Suppose $\text{ProgramPart}(\text{Relocated}(I,n)) \subseteq s_2$ and $\mathbf{IC}_{(s_2)} = \text{insloc}(n)$ and $s_1\restriction(\text{Int-Locations}\cup\text{FinSeq-Locations}) = s_2\restriction(\text{Int-Locations}\cup\text{FinSeq-Locations})$. Let i be a natural number. Then $\mathbf{IC}_{(\text{Computation}(s_1))(i)} + n = \mathbf{IC}_{(\text{Computation}(s_2))(i)}$ and $\text{IncAddr}(\text{CurInstr}((\text{Computation}(s_1))(i)), n) = \text{CurInstr}((\text{Computation}(s_2))(i))$ and $(\text{Computation}(s_1))(i)\restriction(\text{Int-Locatio}$ $(\text{Computation}(s_2))(i)\restriction(\text{Int-Locations}\cup\text{FinSeq-Locations})$.

(12) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, i be a keeping 0 parahalting instruction of $\mathbf{SCM}_{\mathrm{FSA}}$, J be a parahalting macro instruction, and a be an integer location. Then $(\text{IExec}(i; J, s))(a) = (\text{IExec}(J, \text{Exec}(i, \text{Initialize}(s))))(a)$.

(13) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, i be a keeping 0 parahalting instruction of $\mathbf{SCM}_{\mathrm{FSA}}$, J be a parahalting macro instruction, and f be a finite sequence location. Then $(\text{IExec}(i; J, s))(f) = (\text{IExec}(J, \text{Exec}(i, \text{Initialize}(s))))(f)$.

Let a be an integer location and let I, J be macro instructions. The functor **if** $a = 0$ **then** I **else** J yields a macro instruction and is defined by:

(Def. 1) **if** $a = 0$ **then** I **else** $J = (\textbf{if } a = 0 \textbf{ goto } \text{insloc}(\text{card}\, J + 3)); J; \text{Goto}(\text{insloc}(\text{card}\, I + 1)); I; \text{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

The functor **if** $a > 0$ **then** I **else** J yielding a macro instruction is defined as follows:

(Def. 2) **if** $a > 0$ **then** I **else** $J = (\textbf{if } a > 0 \textbf{ goto } \text{insloc}(\text{card}\, J + 3)); J; \text{Goto}(\text{insloc}(\text{card}\, I + 1)); I; \text{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

Let a be an integer location and let I, J be macro instructions. The functor **if** $a < 0$ **then** I **else** J yielding a macro instruction is defined by:

(Def. 3) **if** $a < 0$ **then** I **else** $J =$ **if** $a = 0$ **then** J **else** (**if** $a > 0$ **then** J **else** I).

One can prove the following propositions:

(14) For all macro instructions I, J and for every integer location a holds $\text{card}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J) = \text{card}\, I + \text{card}\, J + 4$.

(15) For all macro instructions I, J and for every integer location a holds $\text{card}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J) = \text{card}\, I + \text{card}\, J + 4$.

(16) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) = 0$ and I is closed on s and halting on s. Then **if** $a = 0$ **then** I **else** J is closed on s and **if** $a = 0$ **then** I **else** J is halting on s.

(17) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) = 0$ and I is closed on $\text{Initialize}(s)$ and halting on $\text{Initialize}(s)$. Then $\text{IExec}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J, s) = \text{IExec}(I, s) +\cdot\, \text{Start-At}(\text{insloc}(\text{card}\, I + \text{card}\, J + 3))$.

(18) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) \neq 0$ and J is closed on s and halting on s. Then **if** $a = 0$ **then** I **else** J is closed on s and **if** $a = 0$ **then** I **else** J is halting on s.

(19) Let I, J be macro instructions, a be a read-write integer location, and s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $s(a) \neq 0$ and J is closed on $\text{Initialize}(s)$ and halting on $\text{Initialize}(s)$. Then $\text{IExec}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J, s) = \text{IExec}(J, s) +\cdot\, \text{Start-At}(\text{insloc}(\text{card}\, I + \text{card}\, J + 3))$.

(20) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be parahalting macro instructions, and a be a read-write integer location. Then **if** $a = 0$ **then** I **else** J is parahalting and if $s(a) = 0$, then $\text{IExec}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J, s) = \text{IExec}(I, s) +\cdot\, \text{Start-At}(\text{insloc}(\text{card}\, I + \text{card}\, J + 3))$ and if $s(a) \neq 0$, then $\text{IExec}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J, s) = \text{IExec}(J, s) +\cdot\, \text{Start-At}(\text{insloc}(\text{card}\, I + \text{card}\, J + 3))$.

(21) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be parahalting macro instructions, and a be a read-write integer location. Then

(i) $\mathbf{IC}_{\mathrm{IExec}(\textbf{if } a=0 \textbf{ then } I \textbf{ else } J,s)} = \mathrm{insloc}(\mathrm{card}\, I + \mathrm{card}\, J + 3)$,

(ii) if $s(a) = 0$, then for every integer location d holds $(\mathrm{IExec}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J, s))(d) = (\mathrm{IExec}(I,s))(d)$ and for every finite sequence location f holds $(\mathrm{IExec}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J, s))(f) = (\mathrm{IExec}(I,s))(f)$, and

(iii) if $s(a) \neq 0$, then for every integer location d holds $(\mathrm{IExec}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J, s))(d) = (\mathrm{IExec}(J,s))(d)$ and for every finite sequence location f holds $(\mathrm{IExec}(\textbf{if } a = 0 \textbf{ then } I \textbf{ else } J, s))(f) = (\mathrm{IExec}(J,s))(f)$.

(22) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) > 0$ and I is closed on s and halting on s. Then **if** $a > 0$ **then** I **else** J is closed on s and **if** $a > 0$ **then** I **else** J is halting on s.

(23) Let I, J be macro instructions, a be a read-write integer location, and s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $s(a) > 0$ and I is closed on Initialize(s) and halting on Initialize(s). Then $\mathrm{IExec}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J, s) = \mathrm{IExec}(I,s) +\cdot \text{Start-At}(\mathrm{insloc}(\mathrm{card}\, I + \mathrm{card}\, J + 3))$.

(24) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) \leq 0$ and J is closed on s and halting on s. Then **if** $a > 0$ **then** I **else** J is closed on s and **if** $a > 0$ **then** I **else** J is halting on s.

(25) Let I, J be macro instructions, a be a read-write integer location, and s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $s(a) \leq 0$ and J is closed on Initialize(s) and halting on Initialize(s). Then $\mathrm{IExec}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J, s) = \mathrm{IExec}(J,s) +\cdot \text{Start-At}(\mathrm{insloc}(\mathrm{card}\, I + \mathrm{card}\, J + 3))$.

(26) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be parahalting macro instructions, and a be a read-write integer location. Then **if** $a > 0$ **then** I **else** J is parahalting and if $s(a) > 0$, then $\mathrm{IExec}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J, s) = \mathrm{IExec}(I,s) +\cdot \text{Start-At}(\mathrm{insloc}(\mathrm{card}\, I + \mathrm{card}\, J + 3))$ and if $s(a) \leq 0$, then $\mathrm{IExec}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J, s) = \mathrm{IExec}(J,s) +\cdot \text{Start-At}(\mathrm{insloc}(\mathrm{card}\, I + \mathrm{card}\, J + 3))$.

(27) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be parahalting macro instructions, and a be a read-write integer location. Then

(i) $\mathbf{IC}_{\mathrm{IExec}(\textbf{if } a>0 \textbf{ then } I \textbf{ else } J,s)} = \mathrm{insloc}(\mathrm{card}\, I + \mathrm{card}\, J + 3)$,

(ii) if $s(a) > 0$, then for every integer location d holds $(\mathrm{IExec}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J, s))(d) = (\mathrm{IExec}(I,s))(d)$ and for every finite sequence location f holds $(\mathrm{IExec}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J, s))(f) = (\mathrm{IExec}(I,s))(f)$, and

(iii) if $s(a) \leq 0$, then for every integer location d holds $(\mathrm{IExec}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J, s))(d) = (\mathrm{IExec}(J,s))(d)$ and for every finite sequence location f holds $(\mathrm{IExec}(\textbf{if } a > 0 \textbf{ then } I \textbf{ else } J, s))(f) = (\mathrm{IExec}(J,s))(f)$.

(28) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) < 0$ and I is closed on s and halting on s. Then **if** $a < 0$ **then** I **else** J is closed on s and **if** $a < 0$ **then** I **else** J is halting on s.

(29) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) < 0$ and I is closed on Initialize(s) and halting on Initialize(s). Then $\mathrm{IExec}(\textbf{if } a < 0 \textbf{ then } I \textbf{ else } J, s) = \mathrm{IExec}(I,s) +\cdot \text{Start-At}(\mathrm{insloc}(\mathrm{card}\, I + \mathrm{card}\, J + \mathrm{card}\, J + 7))$.

(30) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) = 0$ and J is closed on s and halting on s. Then **if** $a < 0$ **then** I **else** J is closed on s and **if** $a < 0$ **then** I **else** J is halting on s.

(31) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) = 0$ and J is closed on Initialize(s) and halting on Initialize(s). Then $\mathrm{IExec}(\textbf{if } a < 0 \textbf{ then } I \textbf{ else } J, s) = \mathrm{IExec}(J,s) +\cdot \text{Start-At}(\mathrm{insloc}(\mathrm{card}\, I + \mathrm{card}\, J + \mathrm{card}\, J + 7))$.

(32) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) > 0$ and J is closed on s and halting on s. Then **if** $a < 0$ **then** I **else** J is closed on s and **if** $a < 0$ **then** I **else** J is halting on s.

(33) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be macro instructions, and a be a read-write integer location. Suppose $s(a) > 0$ and J is closed on Initialize(s) and halting on Initialize(s). Then IExec(**if** $a < 0$ **then** I **else** $J, s) =$ IExec$(J, s)+\cdot$ Start-At(insloc(card$I +$ card$J +$ card$J + 7)$).

(34) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I, J be parahalting macro instructions, and a be a read-write integer location. Then

(i) **if** $a < 0$ **then** I **else** J is parahalting,

(ii) if $s(a) < 0$, then IExec(**if** $a < 0$ **then** I **else** $J, s) =$ IExec$(I, s)+\cdot$ Start-At(insloc(card$I +$ card$J +$ card$J + 7)$), and

(iii) if $s(a) \geq 0$, then IExec(**if** $a < 0$ **then** I **else** $J, s) =$ IExec$(J, s)+\cdot$ Start-At(insloc(card$I +$ card$J +$ card$J + 7)$).

Let I, J be parahalting macro instructions and let a be a read-write integer location. One can verify that **if** $a = 0$ **then** I **else** J is parahalting and **if** $a > 0$ **then** I **else** J is parahalting.

Let a, b be integer locations and let I, J be macro instructions. The functor **if** $a = b$ **then** I **else** J yields a macro instruction and is defined by:

(Def. 4) **if** $a = b$ **then** I **else** $J =$ SubFrom(a, b); (**if** $a = 0$ **then** I **else** J).

The functor **if** $a > b$ **then** I **else** J yielding a macro instruction is defined by:

(Def. 5) **if** $a > b$ **then** I **else** $J =$ SubFrom(a, b); (**if** $a > 0$ **then** I **else** J).

We introduce **if** $b < a$ **then** I **else** J as a synonym of **if** $a > b$ **then** I **else** J.

Let I, J be parahalting macro instructions and let a, b be read-write integer locations. One can check that **if** $a = b$ **then** I **else** J is parahalting and **if** $a > b$ **then** I **else** J is parahalting.

We now state several propositions:

(35) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every macro instruction I holds Result$(s+\cdot$ Initialized$(I))\restriction$(Int-Locations$\cup$FinSe IExec$(I, s)\restriction$(Int-Locations$\cup$FinSeq-Locations).

(36) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, and a be an integer location. Then Result$(s+\cdot$ Initialized$(I))$ and IExec(I, s) are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

(37) Let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$, i be an instruction of $\mathbf{SCM}_{\mathrm{FSA}}$, and a be an integer location. Suppose that

(i) for every integer location b such that $a \neq b$ holds $s_1(b) = s_2(b)$,

(ii) for every finite sequence location f holds $s_1(f) = s_2(f)$,

(iii) i does not refer a, and

(iv) $\mathbf{IC}_{(s_1)} = \mathbf{IC}_{(s_2)}$.

Then

(v) for every integer location b such that $a \neq b$ holds $(\mathrm{Exec}(i, s_1))(b) = (\mathrm{Exec}(i, s_2))(b)$,

(vi) for every finite sequence location f holds $(\mathrm{Exec}(i, s_1))(f) = (\mathrm{Exec}(i, s_2))(f)$, and

(vii) $\mathbf{IC}_{\mathrm{Exec}(i, s_1)} = \mathbf{IC}_{\mathrm{Exec}(i, s_2)}$.

(38) Let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, and a be an integer location. Suppose that

(i) I does not refer a,

(ii) for every integer location b such that $a \neq b$ holds $s_1(b) = s_2(b)$,

(iii) for every finite sequence location f holds $s_1(f) = s_2(f)$, and

(iv) I is closed on s_1 and halting on s_1.

Let k be a natural number. Then

(v) for every integer location b such that $a \neq b$ holds $(\text{Computation}(s_1+\cdot(I+\cdot\text{Start-At}(\text{insloc}(0)))))(k)(b) = (\text{Computation}(s_2+\cdot(I+\cdot\text{Start-At}(\text{insloc}(0)))))(k)(b)$,

(vi) for every finite sequence location f holds $(\text{Computation}(s_1+\cdot(I+\cdot\text{Start-At}(\text{insloc}(0)))))(k)(f) = (\text{Computation}(s_2+\cdot(I+\cdot\text{Start-At}(\text{insloc}(0)))))(k)(f)$,

(vii) $\mathbf{IC}_{(\text{Computation}(s_1+\cdot(I+\cdot\text{Start-At}(\text{insloc}(0)))))(k)} = \mathbf{IC}_{(\text{Computation}(s_2+\cdot(I+\cdot\text{Start-At}(\text{insloc}(0)))))(k)}$, and

(viii) $\text{CurInstr}((\text{Computation}(s_1+\cdot(I+\cdot\text{Start-At}(\text{insloc}(0)))))(k)) = \text{CurInstr}((\text{Computation}(s_2+\cdot(I+\cdot\text{Start-At}(\text{insloc}(0)$

(39) Let s be a state of $\mathbf{SCM}_{\text{FSA}}$, I, J be macro instructions, and l be an instruction-location of $\mathbf{SCM}_{\text{FSA}}$. Then I is closed on s and halting on s if and only if I is closed on $s+\cdot(I+\cdot\text{Start-At}(l))$ and halting on $s+\cdot(I+\cdot\text{Start-At}(l))$.

(40) Let s_1, s_2 be states of $\mathbf{SCM}_{\text{FSA}}$, I be a macro instruction, and a be an integer location. Suppose that

(i) I does not refer a,

(ii) for every integer location b such that $a \neq b$ holds $s_1(b) = s_2(b)$,

(iii) for every finite sequence location f holds $s_1(f) = s_2(f)$, and

(iv) I is closed on s_1 and halting on s_1.

Then I is closed on s_2 and halting on s_2.

(41) Let s_1, s_2 be states of $\mathbf{SCM}_{\text{FSA}}$, I be a macro instruction, and a be an integer location. Suppose that

(i) for every read-write integer location d such that $a \neq d$ holds $s_1(d) = s_2(d)$,

(ii) for every finite sequence location f holds $s_1(f) = s_2(f)$,

(iii) I does not refer a, and

(iv) I is closed on $\text{Initialize}(s_1)$ and halting on $\text{Initialize}(s_1)$.

Then

(v) for every integer location d such that $a \neq d$ holds $(\text{IExec}(I,s_1))(d) = (\text{IExec}(I,s_2))(d)$,

(vi) for every finite sequence location f holds $(\text{IExec}(I,s_1))(f) = (\text{IExec}(I,s_2))(f)$, and

(vii) $\mathbf{IC}_{\text{IExec}(I,s_1)} = \mathbf{IC}_{\text{IExec}(I,s_2)}$.

(42) Let s be a state of $\mathbf{SCM}_{\text{FSA}}$, I, J be parahalting macro instructions, and a, b be read-write integer locations. Suppose I does not refer a and J does not refer a. Then

(i) $\mathbf{IC}_{\text{IExec}(\textbf{if } a=b \textbf{ then } I \textbf{ else } J,s)} = \text{insloc}(\text{card}\, I + \text{card}\, J + 5)$,

(ii) if $s(a) = s(b)$, then for every integer location d such that $a \neq d$ holds $(\text{IExec}(\textbf{if } a = b \textbf{ then } I \textbf{ else } J,s))(d) = (\text{IExec}(I,s))(d)$ and for every finite sequence location f holds $(\text{IExec}(\textbf{if } a = b \textbf{ then } I \textbf{ else } J,s))(f) = (\text{IExec}(I,s))(f)$, and

(iii) if $s(a) \neq s(b)$, then for every integer location d such that $a \neq d$ holds $(\text{IExec}(\textbf{if } a = b \textbf{ then } I \textbf{ else } J,s))(d) = (\text{IExec}(J,s))(d)$ and for every finite sequence location f holds $(\text{IExec}(\textbf{if } a = b \textbf{ then } I \textbf{ else } J,s))(f) = (\text{IExec}(J,s))(f)$.

(43) Let s be a state of $\mathbf{SCM}_{\text{FSA}}$, I, J be parahalting macro instructions, and a, b be read-write integer locations. Suppose I does not refer a and J does not refer a. Then

(i) $\mathbf{IC}_{\text{IExec}(\textbf{if } a>b \textbf{ then } I \textbf{ else } J,s)} = \text{insloc}(\text{card}\, I + \text{card}\, J + 5)$,

(ii) if $s(a) > s(b)$, then for every integer location d such that $a \neq d$ holds $(\text{IExec}(\textbf{if } a > b \textbf{ then } I \textbf{ else } J,s))(d) = (\text{IExec}(I,s))(d)$ and for every finite sequence location f holds $(\text{IExec}(\textbf{if } a > b \textbf{ then } I \textbf{ else } J,s))(f) = (\text{IExec}(I,s))(f)$, and

(iii) if $s(a) \leq s(b)$, then for every integer location d such that $a \neq d$ holds $(\text{IExec}(\textbf{if } a > b \textbf{ then } I \textbf{ else } J,s))(d) = (\text{IExec}(J,s))(d)$ and for every finite sequence location f holds $(\text{IExec}(\textbf{if } a > b \textbf{ then } I \textbf{ else } J,s))(f) = (\text{IExec}(J,s))(f)$.

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