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Conditional Branch Macro Instructions of $\mathbf{SCM}_{\rm FSA}$. Part I

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The articles [14], [5], [10], [20], [9], [11], [8], [4], [6], [12], [15], [13], [19], [16], [17], [7], [18], [2], [3], and [1] provide the notation and terminology for this paper.

One can prove the following propositions:

(2)[1] For all functions f, g and for every set D such that $\operatorname{dom} g$ misses D holds $(f+\cdot g){\restriction}D = f{\restriction}D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(3) For every state s of $\mathbf{SCM}_{\rm FSA}$ holds $\operatorname{dom}(s{\restriction}$the instruction locations of $\mathbf{SCM}_{\rm FSA}) =$ the instruction locations of $\mathbf{SCM}_{\rm FSA}$.

(4) For every state s of $\mathbf{SCM}_{\rm FSA}$ such that s is halting and for every natural number k such that $\text{LifeSpan}(s) \leq k$ holds $\text{CurInstr}((\text{Computation}(s))(k)) = \mathbf{halt}_{\mathbf{SCM}_{\rm FSA}}$.

(5) For every state s of $\mathbf{SCM}_{\rm FSA}$ such that s is halting and for every natural number k such that $\text{LifeSpan}(s) \leq k$ holds $\mathbf{IC}_{(\text{Computation}(s))(k)} = \mathbf{IC}_{(\text{Computation}(s))(\text{LifeSpan}(s))}$.

(6) Let s_1, s_2 be states of $\mathbf{SCM}_{\rm FSA}$. Then s_1 and s_2 are equal outside the instruction locations of $\mathbf{SCM}_{\rm FSA}$ if and only if $\mathbf{IC}_{(s_1)} = \mathbf{IC}_{(s_2)}$ and $s_1{\restriction}(\text{Int-Locations} \cup \text{FinSeq-Locations}) = s_2{\restriction}(\text{Int-Locations} \cup \text{FinSeq-Locations})$.

(7) For every state s of $\mathbf{SCM}_{\rm FSA}$ and for every macro instruction I holds $\mathbf{IC}_{\text{IExec}(I,s)} = \mathbf{IC}_{\text{Result}(s+\cdot \text{Initialized}(I))}$.

(8) For every state s of $\mathbf{SCM}_{\rm FSA}$ and for every macro instruction I holds $\text{Initialize}(s)+\cdot \text{Initialized}(I) = s+\cdot \text{Initialized}(I)$.

(9) For every macro instruction I and for every instruction-location l of $\mathbf{SCM}_{\rm FSA}$ holds $I \subseteq I+\cdot \text{Start-At}(l)$.

(10) For every state s of $\mathbf{SCM}_{\rm FSA}$ and for every instruction-location l of $\mathbf{SCM}_{\rm FSA}$ holds $s{\restriction}(\text{Int-Locations} \cup \text{FinSeq-Locations}) = (s+\cdot \text{Start-At}(l)){\restriction}(\text{Int-Locations} \cup \text{FinSeq-Locations})$.

(11) Let s be a state of $\mathbf{SCM}_{\rm FSA}$, I be a macro instruction, and l be an instruction-location of $\mathbf{SCM}_{\rm FSA}$. Then $s{\restriction}(\text{Int-Locations} \cup \text{FinSeq-Locations}) = (s+\cdot(I+\cdot \text{Start-At}(l))){\restriction}(\text{Int-Locations} \cup \text{FinSeq-Locations})$.

(12) Let s be a state of $\mathbf{SCM}_{\rm FSA}$ and l be an instruction-location of $\mathbf{SCM}_{\rm FSA}$. Then $\operatorname{dom}(s{\restriction}$the instruction locations of $\mathbf{SCM}_{\rm FSA})$ misses $\operatorname{dom} \text{Start-At}(l)$.

[1] The proposition (1) has been removed.

(13) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every macro instruction I holds $s+\cdot\mathrm{Initialized}(I) = \mathrm{Initialize}(s)+\cdot(I+\cdot\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))$.

(14) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I_1, I_2 be macro instructions, and l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. Then $s+\cdot(I_1+\cdot\mathrm{Start\text{-}At}(l))$ and $s+\cdot(I_2+\cdot\mathrm{Start\text{-}At}(l))$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

(15) $\mathrm{dom}(\mathrm{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}) = \{\mathrm{insloc}(0)\}$.

(16) $\mathrm{insloc}(0) \in \mathrm{dom}(\mathrm{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}})$ and $\mathrm{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}(\mathrm{insloc}(0)) = \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$.

(17) $\mathrm{card}(\mathrm{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}) = 1$.

Let P be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and let l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. The functor $\mathrm{Directed}(P,l)$ yields a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined by:

(Def. 1) $\mathrm{Directed}(P,l) = (\mathrm{id}_{\text{the instructions of } \mathbf{SCM}_{\mathrm{FSA}}}+\cdot(\mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}} \dot{\longmapsto} \mathrm{goto}\ l)) \cdot P$.

One can prove the following proposition

(18) For every programmed finite partial state I of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\mathrm{Directed}(I) = \mathrm{Directed}(I,\mathrm{insloc}(\mathrm{card}\,I))$.

Let P be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and let l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. One can check that $\mathrm{Directed}(P,l)$ is halt-free.

Let P be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. Note that $\mathrm{Directed}(P)$ is halt-free.

One can prove the following propositions:

(19) For every programmed finite partial state P of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every instruction-location l of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\mathrm{dom}\,\mathrm{Directed}(P,l) = \mathrm{dom}\,P$.

(20) Let P be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. Then $\mathrm{Directed}(P,l) = P+\cdot(\mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}} \dot{\longmapsto} \mathrm{goto}\ l) \cdot P$.

(21) Let P be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$, and x be a set such that $x \in \mathrm{dom}\,P$. Then

(i) if $P(x) = \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$, then $(\mathrm{Directed}(P,l))(x) = \mathrm{goto}\ l$, and

(ii) if $P(x) \neq \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$, then $(\mathrm{Directed}(P,l))(x) = P(x)$.

(22) Let i be an instruction of $\mathbf{SCM}_{\mathrm{FSA}}$, a be an integer location, and n be a natural number. If i does not destroy a, then $\mathrm{IncAddr}(i,n)$ does not destroy a.

(23) Let P be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, n be a natural number, and a be an integer location. If P does not destroy a, then $\mathrm{ProgramPart}(\mathrm{Relocated}(P,n))$ does not destroy a.

(24) For every good programmed finite partial state P of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number n holds $\mathrm{ProgramPart}(\mathrm{Relocated}(P,n))$ is good.

(25) Let I, J be programmed finite partial states of $\mathbf{SCM}_{\mathrm{FSA}}$ and a be an integer location. Suppose I does not destroy a and J does not destroy a. Then $I+\cdot J$ does not destroy a.

(26) For all good programmed finite partial states I, J of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $I+\cdot J$ is good.

(27) Let I be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$, and a be an integer location. If I does not destroy a, then $\mathrm{Directed}(I,l)$ does not destroy a.

Let I be a good programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and let l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. Observe that $\mathrm{Directed}(I,l)$ is good.

Let I be a good macro instruction. One can verify that $\mathrm{Directed}(I)$ is good.

Let I be a macro instruction and let l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. Note that $\mathrm{Directed}(I,l)$ is initial.

Let I, J be good macro instructions. Observe that I; J is good.

Let l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. The functor $\mathrm{Goto}(l)$ yields a halt-free good macro instruction and is defined as follows:

(Def. 2) $\mathrm{Goto}(l) = \mathrm{insloc}(0) \dot{\longmapsto} \mathrm{goto}\ l$.

Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and let P be an initial finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. We say that P is pseudo-closed on s if and only if the condition (Def. 3) is satisfied.

(Def. 3) There exists a natural number k such that $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot(P+\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(k)} = \mathrm{insloc}(\mathrm{card}\,\mathrm{ProgramPart}(P))$ and for every natural number n such that $n < k$ holds $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot(P+\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(n)} \in \mathrm{dom}\,P$.

Let P be an initial finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. We say that P is pseudo-paraclosed if and only if:

(Def. 4) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ holds P is pseudo-closed on s.

Let us note that there exists a macro instruction which is pseudo-paraclosed.

Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and let P be an initial finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. Let us assume that P is pseudo-closed on s. The functor $\mathrm{pseudo}-\mathrm{LifeSpan}(s,P)$ yields a natural number and is defined by:

(Def. 5) $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot(P+\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(\mathrm{pseudo}-\mathrm{LifeSpan}(s,P))} = \mathrm{insloc}(\mathrm{card}\,\mathrm{ProgramPart}(P))$ and for every natural number n such that $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot(P+\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(n)} \notin \mathrm{dom}\,P$ holds $\mathrm{pseudo}-\mathrm{LifeSpan}(s,P) \leq n$.

One can prove the following propositions:

(28) For all macro instructions I, J and for every set x such that $x \in \mathrm{dom}\,I$ holds $(I;\,J)(x) = (\mathrm{Directed}(I))(x)$.

(29) For every instruction-location l of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\mathrm{card}\,\mathrm{Goto}(l) = 1$.

(30) Let P be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and x be a set such that $x \in \mathrm{dom}\,P$. Then

(i) if $P(x) = \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$, then $(\mathrm{Directed}(P))(x) = \mathrm{goto}\ \mathrm{insloc}(\mathrm{card}\,P)$, and

(ii) if $P(x) \neq \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$, then $(\mathrm{Directed}(P))(x) = P(x)$.

(31) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and P be an initial finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose P is pseudo-closed on s. Let n be a natural number. If $n < \mathrm{pseudo}-\mathrm{LifeSpan}(s,P)$, then $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot(P+\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(n)} \in \mathrm{dom}\,P$ and $\mathrm{CurInstr}((\mathrm{Computation}(s+\cdot(P+\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(n)) \neq \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$.

(32) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I, J be macro instructions. Suppose I is pseudo-closed on s. Let k be a natural number. Suppose $k \leq \mathrm{pseudo}-\mathrm{LifeSpan}(s,I)$. Then $(\mathrm{Computation}(s+\cdot(I+\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(k)$ and $(\mathrm{Computation}(s+\cdot((I;\,J)+\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(k)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

(33) For every programmed finite partial state I of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every instruction-location l of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\mathrm{card}\,\mathrm{Directed}(I,l) = \mathrm{card}\,I$.

(34) For every macro instruction I holds $\mathrm{card}\,\mathrm{Directed}(I) = \mathrm{card}\,I$.

(35) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. Suppose I is closed on s and halting on s. Let k be a natural number. Suppose $k \leq \mathrm{LifeSpan}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0))))$. Then $(\mathrm{Computation}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(k)$ and $(\mathrm{Computation}(s+\cdot(\mathrm{Directed}(I)+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(k)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$ and CurInstr((Computation($s+\cdot$(Directed(I)$+\cdot$ Start-At(insloc(0 $\mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$.

(36) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. Suppose I is closed on s and halting on s. Then $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot(\mathrm{Directed}(I)+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(\mathrm{LifeSpan}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0))))+1)} =$ $\mathrm{insloc}(\mathrm{card}\,I)$ and $(\mathrm{Computation}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(\mathrm{LifeSpan}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))\restriction$(Int-Locatio $(\mathrm{Computation}(s+\cdot(\mathrm{Directed}(I)+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(\mathrm{LifeSpan}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0))))+1)\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{FinSeq\text{-}Locations})$.

(37) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. If I is closed on s and halting on s, then $\mathrm{Directed}(I)$ is pseudo-closed on s.

(38) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. If I is closed on s and halting on s, then $\mathrm{pseudo}-\mathrm{LifeSpan}(s,\mathrm{Directed}(I)) = \mathrm{LifeSpan}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0))))+1$.

(39) Let I be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. If I is halt-free, then $\mathrm{Directed}(I,l) = I$.

(40) For every macro instruction I such that I is halt-free holds $\mathrm{Directed}(I) = I$.

(41) For all macro instructions I, J holds $\mathrm{Directed}(I);\, J = I;\, J$.

(42) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I, J be macro instructions. Suppose I is closed on s and halting on s. Then

(i) for every natural number k such that $k \leq \mathrm{LifeSpan}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0))))$ holds $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot(\mathrm{Directed}(I)+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(k)} = \mathbf{IC}_{(\mathrm{Computation}(s+\cdot((I;\,J)+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(k)}$ and CurInstr((Computation($s+\cdot$(Directed(I)$+\cdot$ Start-At(insloc(0)))))(k)) = CurInstr((Computation($s+\cdot$((I; J)$+\cdot$ Start

(ii) $(\mathrm{Computation}(s+\cdot(\mathrm{Directed}(I)+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(\mathrm{LifeSpan}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0))))+1)\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{FinSeq\text{-}Locations}) =$ (Computation($s+\cdot$((I; J)$+\cdot$ Start-At(insloc(0)))))(LifeSpan($s+\cdot$($I+\cdot$ Start-A $1)\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{FinSeq\text{-}Locations})$, and

(iii) $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot(\mathrm{Directed}(I)+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))))(\mathrm{LifeSpan}(s+\cdot(I+\cdot\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0))))+1)} = \mathbf{IC}$(Computation($s+\cdot$(($I$; J)$+\cdot$ Start-At(insloc

(43) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I, J be macro instructions. Suppose I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$. Then

(i) for every natural number k such that $k \leq \mathrm{LifeSpan}(s+\cdot\,\mathrm{Initialized}(I))$ holds $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(\mathrm{Directed}(I))))(k)} = \mathbf{IC}_{(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(I;\,J)))(k)}$ and CurInstr((Computation($s+\cdot$ Initialized(D CurInstr((Computation($s+\cdot$ Initialized(I; J)))(k)),

(ii) $(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(\mathrm{Directed}(I))))(\mathrm{LifeSpan}(s+\cdot\,\mathrm{Initialized}(I))+1)\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{FinSeq\text{-}Locations})$ $(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(I;\,J)))(\mathrm{LifeSpan}(s+\cdot\,\mathrm{Initialized}(I))+1)\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{FinSeq\text{-}Locations})$, and

(iii) $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(\mathrm{Directed}(I))))(\mathrm{LifeSpan}(s+\cdot\,\mathrm{Initialized}(I))+1)} = \mathbf{IC}$(Computation($s+\cdot$ Initialized(I; J)))(LifeSpan($s+\cdot$ Initialized(I))

(44) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. Suppose I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$. Let k be a natural number. Suppose $k \leq \mathrm{LifeSpan}(s+\cdot\,\mathrm{Initialized}(I))$. Then $(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(I)))(k)$ and $(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(\mathrm{Directed}(I))))(k)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$ and $\mathrm{CurInstr}((\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(\mathrm{Directed}(I))))(k)) \neq \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$.

(45) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. Suppose I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$. Then $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(\mathrm{Directed}(I))))(\mathrm{LifeSpan}(s+\cdot\,\mathrm{Initialized}(I))+1)} =$ $\mathrm{insloc}(\mathrm{card}\,I)$ and $(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(I)))(\mathrm{LifeSpan}(s+\cdot\,\mathrm{Initialized}(I)))\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{FinSeq\text{-}Locations})$ $(\mathrm{Computation}(s+\cdot\,\mathrm{Initialized}(\mathrm{Directed}(I))))(\mathrm{LifeSpan}(s+\cdot\,\mathrm{Initialized}(I))+1)\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{FinSeq\text{-}Locations})$.

(46) Let I be a macro instruction and s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose I is closed on s and halting on s. Then $I;\, \mathrm{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is closed on s and $I;\, \mathrm{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is halting on s.

(47) For every instruction-location l of $\mathbf{SCM}_{\rm FSA}$ holds $\mathrm{insloc}(0) \in \mathrm{dom}\,\mathrm{Goto}(l)$ and $(\mathrm{Goto}(l))(\mathrm{insloc}(0)) = \mathrm{goto}\ l$.

(48) Let N be a set with non empty elements, S be a definite non empty non void AMI over N, I be a programmed finite partial state of S, and x be a set. If $x \in \mathrm{dom}\,I$, then $I(x)$ is an instruction of S.

(49) Let I be a programmed finite partial state of $\mathbf{SCM}_{\rm FSA}$, x be a set, and k be a natural number. If $x \in \mathrm{dom}\,\mathrm{ProgramPart}(\mathrm{Relocated}(I,k))$, then $(\mathrm{ProgramPart}(\mathrm{Relocated}(I,k)))(x) = (\mathrm{Relocated}(I,k))(x)$.

(50) For every programmed finite partial state I of $\mathbf{SCM}_{\rm FSA}$ and for every natural number k holds $\mathrm{ProgramPart}(\mathrm{Relocated}(\mathrm{Directed}(I),k)) = \mathrm{Directed}(\mathrm{ProgramPart}(\mathrm{Relocated}(I,k)),\mathrm{insloc}(\mathrm{card}\,I+k))$.

(51) Let I, J be programmed finite partial states of $\mathbf{SCM}_{\rm FSA}$ and l be an instruction-location of $\mathbf{SCM}_{\rm FSA}$. Then $\mathrm{Directed}(I{+\cdot}J,l) = \mathrm{Directed}(I,l){+\cdot}\,\mathrm{Directed}(J,l)$.

(52) For all macro instructions I, J holds $\mathrm{Directed}(I;\ J) = I;\ \mathrm{Directed}(J)$.

(53) Let I be a macro instruction and s be a state of $\mathbf{SCM}_{\rm FSA}$. If I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$, then $\mathbf{IC}_{(\mathrm{Computation}(s{+\cdot}\,\mathrm{Initialized}(I;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}})))(\mathrm{LifeSpan}(s{+\cdot}\,\mathrm{Initialized}(I))+1)} = \mathrm{insloc}(\mathrm{card}\,I)$.

(54) Let I be a macro instruction and s be a state of $\mathbf{SCM}_{\rm FSA}$. Suppose I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$. Then $(\mathrm{Computation}(s{+\cdot}\,\mathrm{Initialized}(I)))(\mathrm{LifeSpan}(s{+\cdot}\,\mathrm{Initialized}(I)))\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{Fin}$ $(\mathrm{Computation}(s{+\cdot}\,\mathrm{Initialized}(I;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}})))(\mathrm{LifeSpan}(s{+\cdot}\,\mathrm{Initialized}(I))+1)\restriction(\mathrm{Int\text{-}Locations}\cup\mathrm{FinSeq\text{-}Locations})$.

(55) Let I be a macro instruction and s be a state of $\mathbf{SCM}_{\rm FSA}$. If I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$, then $s{+\cdot}\,\mathrm{Initialized}(I;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}})$ is halting.

(56) Let I be a macro instruction and s be a state of $\mathbf{SCM}_{\rm FSA}$. If I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$, then $\mathrm{LifeSpan}(s{+\cdot}\,\mathrm{Initialized}(I;\mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}})) = \mathrm{LifeSpan}(s{+\cdot}\,\mathrm{Initialized}(I))+1$.

(57) Let s be a state of $\mathbf{SCM}_{\rm FSA}$ and I be a macro instruction. If I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$, then $\mathrm{IExec}(I;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}},s) = \mathrm{IExec}(I,s){+\cdot}\,\mathrm{Start\text{-}At}(\mathrm{insloc}(\mathrm{card}\,I))$.

(58) Let I, J be macro instructions and s be a state of $\mathbf{SCM}_{\rm FSA}$. Suppose I is closed on s and halting on s. Then $I;\ \mathrm{Goto}(\mathrm{insloc}(\mathrm{card}\,J+1));\ J;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}}$ is closed on s and $I;\ \mathrm{Goto}(\mathrm{insloc}(\mathrm{card}\,J+1));\ J;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}}$ is halting on s.

(59) Let I, J be macro instructions and s be a state of $\mathbf{SCM}_{\rm FSA}$. If I is closed on s and halting on s, then $s{+\cdot}((I;\ \mathrm{Goto}(\mathrm{insloc}(\mathrm{card}\,J+1));\ J;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}}){+\cdot}\,\mathrm{Start\text{-}At}(\mathrm{insloc}(0)))$ is halting.

(60) Let I, J be macro instructions and s be a state of $\mathbf{SCM}_{\rm FSA}$. If I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$, then $s{+\cdot}\,\mathrm{Initialized}(I;\ \mathrm{Goto}(\mathrm{insloc}(\mathrm{card}\,J+1));\ J;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}})$ is halting.

(61) Let I, J be macro instructions and s be a state of $\mathbf{SCM}_{\rm FSA}$. If I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$, then $\mathbf{IC}_{\mathrm{IExec}(I;\ \mathrm{Goto}(\mathrm{insloc}(\mathrm{card}\,J+1));\ J;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}},s)} = \mathrm{insloc}(\mathrm{card}\,I+\mathrm{card}\,J+1)$.

(62) Let I, J be macro instructions and s be a state of $\mathbf{SCM}_{\rm FSA}$. Suppose I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$. Then $\mathrm{IExec}(I;\ \mathrm{Goto}(\mathrm{insloc}(\mathrm{card}\,J+1));\ J;\ \mathrm{Stop}_{\mathbf{SCM}_{\rm FSA}},s) = \mathrm{IExec}(I,s){+\cdot}\,\mathrm{Start\text{-}At}(\mathrm{insloc}(\mathrm{card}\,I+\mathrm{card}\,J+1))$.

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