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Schemes of Existence of Some Types of Functions

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Summary. We prove some useful schemes of existence of real sequences, partial functions from a domain into a domain, partial functions from a set to a set and functions from a domain into a domain. At the beginning we prove some related auxiliary theorems related to the article [1].

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The articles [8], [5], [10], [9], [1], [11], [2], [12], [4], [3], [7], and [6] provide the notation and terminology for this paper.

We use the following convention: x, y are sets, n, m are natural numbers, and r is a real number.

Next we state four propositions:

(1) For every n there exists m such that $n = 2 \cdot m$ or $n = 2 \cdot m + 1$.

(2) For every n there exists m such that $n = 3 \cdot m$ or $n = 3 \cdot m + 1$ or $n = 3 \cdot m + 2$.

(3) For every n there exists m such that $n = 4 \cdot m$ or $n = 4 \cdot m + 1$ or $n = 4 \cdot m + 2$ or $n = 4 \cdot m + 3$.

(4) For every n there exists m such that $n = 5 \cdot m$ or $n = 5 \cdot m + 1$ or $n = 5 \cdot m + 2$ or $n = 5 \cdot m + 3$ or $n = 5 \cdot m + 4$.

In this article we present several logical schemes. The scheme *ExRealSubseq* deals with a sequence $\mathcal{A}$ of real numbers and a unary predicate $\mathcal{P}$, and states that:

There exists a sequence q of real numbers such that
(i) q is a subsequence of $\mathcal{A}$,
(ii) for every n holds $\mathcal{P}[q(n)]$, and
(iii) for every n such that for every r such that $r = \mathcal{A}(n)$ holds $\mathcal{P}[r]$ there exists m such that $\mathcal{A}(n) = q(m)$

provided the following condition is satisfied:

- For every n there exists m such that $n \leq m$ and $\mathcal{P}[\mathcal{A}(m)]$.

The scheme *ExRealSeq2* deals with two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding real numbers, and states that:

There exists a sequence s of real numbers such that for every n holds $s(2 \cdot n) = \mathcal{F}(n)$ and $s(2 \cdot n + 1) = \mathcal{G}(n)$

for all values of the parameter.

The scheme *ExRealSeq3* deals with three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding real numbers, and states that:

There exists a sequence s of real numbers such that for every n holds $s(3 \cdot n) = \mathcal{F}(n)$ and $s(3 \cdot n + 1) = \mathcal{G}(n)$ and $s(3 \cdot n + 2) = \mathcal{H}(n)$

for all values of the parameter.

The scheme *ExRealSeq4* deals with four unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{I}$ yielding real numbers, and states that:

There exists a sequence s of real numbers such that for every n holds $s(4\cdot n) = \mathcal{F}(n)$ and $s(4\cdot n+1) = \mathcal{G}(n)$ and $s(4\cdot n+2) = \mathcal{H}(n)$ and $s(4\cdot n+3) = \mathcal{I}(n)$

for all values of the parameter.

The scheme *ExRealSeq5* deals with five unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, $\mathcal{I}$, and $\mathcal{J}$ yielding real numbers, and states that:

There exists a sequence s of real numbers such that for every n holds $s(5\cdot n) = \mathcal{F}(n)$ and $s(5\cdot n+1) = \mathcal{G}(n)$ and $s(5\cdot n+2) = \mathcal{H}(n)$ and $s(5\cdot n+3) = \mathcal{I}(n)$ and $s(5\cdot n+4) = \mathcal{J}(n)$

for all values of the parameter.

The scheme *PartFuncExD2* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{B}$, and two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \mathrm{dom}\, f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$, and

(ii) for every element c of $\mathcal{A}$ such that $c \in \mathrm{dom}\, f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$

provided the parameters meet the following condition:

- For every element c of $\mathcal{A}$ such that $\mathcal{P}[c]$ holds not $\mathcal{Q}[c]$.

The scheme *PartFuncExD2'* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{B}$, and two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \mathrm{dom}\, f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$, and

(ii) for every element c of $\mathcal{A}$ such that $c \in \mathrm{dom}\, f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$

provided the following condition is satisfied:

- For every element c of $\mathcal{A}$ such that $\mathcal{P}[c]$ and $\mathcal{Q}[c]$ holds $\mathcal{F}(c) = \mathcal{G}(c)$.

The scheme *PartFuncExD2"* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{B}$, and a unary predicate $\mathcal{P}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that f is total and for every element c of $\mathcal{A}$ such that $c \in \mathrm{dom}\, f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if not $\mathcal{P}[c]$, then $f(c) = \mathcal{G}(c)$

for all values of the parameters.

The scheme *PartFuncExD3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \mathrm{dom}\, f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$, and

(ii) for every element c of $\mathcal{A}$ such that $c \in \mathrm{dom}\, f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ and if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$

provided the following condition is satisfied:

- For every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$ and if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$ and if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$.

The scheme *PartFuncExD3'* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \mathrm{dom}\, f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$, and

(ii) for every element c of $\mathcal{A}$ such that $c \in \mathrm{dom}\, f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ and if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$

provided the following condition is met:

- Let c be an element of $\mathcal{A}$. Then
  (i) if $\mathcal{P}[c]$ and $\mathcal{Q}[c]$, then $\mathcal{F}(c) = \mathcal{G}(c)$,
  (ii) if $\mathcal{P}[c]$ and $\mathcal{R}[c]$, then $\mathcal{F}(c) = \mathcal{H}(c)$, and
  (iii) if $\mathcal{Q}[c]$ and $\mathcal{R}[c]$, then $\mathcal{G}(c) = \mathcal{H}(c)$.

The scheme *PartFuncExD4* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, four unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{I}$ yielding elements of $\mathcal{B}$, and four unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$ or $\mathcal{S}[c]$, and
(ii) for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ and if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$ and if $\mathcal{S}[c]$, then $f(c) = \mathcal{I}(c)$

provided the following requirement is met:

- Let c be an element of $\mathcal{A}$. Then
  (i) if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$,
  (ii) if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$,
  (iii) if $\mathcal{P}[c]$, then not $\mathcal{S}[c]$,
  (iv) if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$,
  (v) if $\mathcal{Q}[c]$, then not $\mathcal{S}[c]$, and
  (vi) if $\mathcal{R}[c]$, then not $\mathcal{S}[c]$.

The scheme *PartFuncExS2* deals with sets $\mathcal{A}$, $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding sets, and two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that
(i) for every x holds $x \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$, and
(ii) for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ and if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$

provided the parameters meet the following requirements:

- For every x such that $x \in \mathcal{A}$ holds if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$, and
- For every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$.

The scheme *PartFuncExS3* deals with sets $\mathcal{A}$, $\mathcal{B}$, three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding sets, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that
(i) for every x holds $x \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$ or $\mathcal{R}[x]$, and
(ii) for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ and if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$ and if $\mathcal{R}[x]$, then $f(x) = \mathcal{H}(x)$

provided the parameters meet the following requirements:

- For every x such that $x \in \mathcal{A}$ holds if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$ and if $\mathcal{P}[x]$, then not $\mathcal{R}[x]$ and if $\mathcal{Q}[x]$, then not $\mathcal{R}[x]$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$, and
- For every x such that $x \in \mathcal{A}$ and $\mathcal{R}[x]$ holds $\mathcal{H}(x) \in \mathcal{B}$.

The scheme *PartFuncExS4* deals with sets $\mathcal{A}$, $\mathcal{B}$, four unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{I}$ yielding sets, and four unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that
(i) for every x holds $x \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$ or $\mathcal{R}[x]$ or $\mathcal{S}[x]$, and
(ii) for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ and if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$ and if $\mathcal{R}[x]$, then $f(x) = \mathcal{H}(x)$ and if $\mathcal{S}[x]$, then $f(x) = \mathcal{I}(x)$

provided the parameters satisfy the following conditions:

- Let given x such that $x \in \mathcal{A}$. Then
  (i) if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$,
  (ii) if $\mathcal{P}[x]$, then not $\mathcal{R}[x]$,
  (iii) if $\mathcal{P}[x]$, then not $\mathcal{S}[x]$,
  (iv) if $\mathcal{Q}[x]$, then not $\mathcal{R}[x]$,
  (v) if $\mathcal{Q}[x]$, then not $\mathcal{S}[x]$, and
  (vi) if $\mathcal{R}[x]$, then not $\mathcal{S}[x]$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{R}[x]$ holds $\mathcal{H}(x) \in \mathcal{B}$, and
- For every x such that $x \in \mathcal{A}$ and $\mathcal{S}[x]$ holds $\mathcal{I}(x) \in \mathcal{B}$.

The scheme *PartFuncExC D2* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, two binary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{C}$, and two binary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $[: \mathcal{A}, \mathcal{B} :]$ to $\mathcal{C}$ such that

(i) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d\rangle \in \operatorname{dom} f$ iff $\mathcal{P}[c,d]$ or $Q[c,d]$, and

(ii) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\langle c, d\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{F}(c,d)$ and if $Q[c,d]$, then $f(\langle c, d\rangle) = \mathcal{G}(c,d)$

provided the following requirement is met:

- For every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\mathcal{P}[c,d]$ holds not $Q[c,d]$.

The scheme *PartFuncExC D3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, three binary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{C}$, and three binary predicates $\mathcal{P}$, Q, $\mathcal{R}$, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that

(i) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d\rangle \in \operatorname{dom} f$ iff $\mathcal{P}[c,d]$ or $Q[c,d]$ or $\mathcal{R}[c,d]$, and

(ii) for every element c of $\mathcal{A}$ and for every element r of $\mathcal{B}$ such that $\langle c, r\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c,r]$, then $f(\langle c, r\rangle) = \mathcal{F}(c,r)$ and if $Q[c,r]$, then $f(\langle c, r\rangle) = \mathcal{G}(c,r)$ and if $\mathcal{R}[c,r]$, then $f(\langle c, r\rangle) = \mathcal{H}(c,r)$

provided the following requirement is met:

- Let c be an element of $\mathcal{A}$ and s be an element of $\mathcal{B}$. Then
  (i) if $\mathcal{P}[c,s]$, then not $Q[c,s]$,
  (ii) if $\mathcal{P}[c,s]$, then not $\mathcal{R}[c,s]$, and
  (iii) if $Q[c,s]$, then not $\mathcal{R}[c,s]$.

The scheme *PartFuncExC S2* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, two binary functors $\mathcal{F}$ and $\mathcal{G}$ yielding sets, and two binary predicates $\mathcal{P}$, Q, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that

(i) for all x, y holds $\langle x, y\rangle \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $y \in \mathcal{B}$ but $\mathcal{P}[x,y]$ or $Q[x,y]$, and

(ii) for all x, y such that $\langle x, y\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[x,y]$, then $f(\langle x, y\rangle) = \mathcal{F}(x,y)$ and if $Q[x,y]$, then $f(\langle x, y\rangle) = \mathcal{G}(x,y)$

provided the following conditions are satisfied:

- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x,y]$, then not $Q[x,y]$,
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x,y]$ holds $\mathcal{F}(x,y) \in \mathcal{C}$, and
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $Q[x,y]$ holds $\mathcal{G}(x,y) \in \mathcal{C}$.

The scheme *PartFuncExC S3* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, three binary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding sets, and three binary predicates $\mathcal{P}$, Q, $\mathcal{R}$, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that

(i) for all x, y holds $\langle x, y\rangle \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $y \in \mathcal{B}$ but $\mathcal{P}[x,y]$ or $Q[x,y]$ or $\mathcal{R}[x,y]$, and

(ii) for all x, y such that $\langle x, y\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[x,y]$, then $f(\langle x, y\rangle) = \mathcal{F}(x,y)$ and if $Q[x,y]$, then $f(\langle x, y\rangle) = \mathcal{G}(x,y)$ and if $\mathcal{R}[x,y]$, then $f(\langle x, y\rangle) = \mathcal{H}(x,y)$

provided the parameters meet the following requirements:

- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x,y]$, then not $Q[x,y]$ and if $\mathcal{P}[x,y]$, then not $\mathcal{R}[x,y]$ and if $Q[x,y]$, then not $\mathcal{R}[x,y]$,
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x,y]$, then $\mathcal{F}(x,y) \in \mathcal{C}$,
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $Q[x,y]$, then $\mathcal{G}(x,y) \in \mathcal{C}$, and
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{R}[x,y]$, then $\mathcal{H}(x,y) \in \mathcal{C}$.

The scheme *ExFuncD3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, Q, $\mathcal{R}$, and states that:

There exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds

(i) if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$,

(ii) if $Q[c]$, then $f(c) = \mathcal{G}(c)$, and

(iii) if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$

provided the parameters satisfy the following conditions:

- For every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then not $Q[c]$ and if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$ and if $Q[c]$, then not $\mathcal{R}[c]$, and
- For every element c of $\mathcal{A}$ holds $\mathcal{P}[c]$ or $Q[c]$ or $\mathcal{R}[c]$.

The scheme *ExFuncD4* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, four unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and I yielding elements of $\mathcal{B}$, and four unary predicates $\mathcal{P}$, Q, $\mathcal{R}$, $\mathcal{S}$, and states that:

There exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds

(i) if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$,
(ii) if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$,
(iii) if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$, and
(iv) if $\mathcal{S}[c]$, then $f(c) = \mathcal{I}(c)$

provided the parameters satisfy the following conditions:

- Let c be an element of $\mathcal{A}$. Then
  (i) if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$,
  (ii) if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$,
  (iii) if $\mathcal{P}[c]$, then not $\mathcal{S}[c]$,
  (iv) if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$,
  (v) if $\mathcal{Q}[c]$, then not $\mathcal{S}[c]$, and
  (vi) if $\mathcal{R}[c]$, then not $\mathcal{S}[c]$,
  and
- For every element c of $\mathcal{A}$ holds $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$ or $\mathcal{S}[c]$.

The scheme *FuncExC D2* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, two binary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{C}$, and a binary predicate $\mathcal{P}$, and states that:

There exists a function f from $[:\mathcal{A}, \mathcal{B}:]$ into $\mathcal{C}$ such that for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds
(i) if $\mathcal{P}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{F}(c,d)$, and
(ii) if not $\mathcal{P}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{G}(c,d)$

for all values of the parameters.

The scheme *FuncExC D3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, three binary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{C}$, and three binary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a function f from $[:\mathcal{A}, \mathcal{B}:]$ into $\mathcal{C}$ such that
(i) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d\rangle \in \operatorname{dom} f$ iff $\mathcal{P}[c,d]$ or $\mathcal{Q}[c,d]$ or $\mathcal{R}[c,d]$, and
(ii) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\langle c, d\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{F}(c,d)$ and if $\mathcal{Q}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{G}(c,d)$ and if $\mathcal{R}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{H}(c,d)$

provided the parameters meet the following requirements:

- Let c be an element of $\mathcal{A}$ and d be an element of $\mathcal{B}$. Then
  (i) if $\mathcal{P}[c,d]$, then not $\mathcal{Q}[c,d]$,
  (ii) if $\mathcal{P}[c,d]$, then not $\mathcal{R}[c,d]$, and
  (iii) if $\mathcal{Q}[c,d]$, then not $\mathcal{R}[c,d]$,
  and
- For every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\mathcal{P}[c,d]$ or $\mathcal{Q}[c,d]$ or $\mathcal{R}[c,d]$.

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