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Average Value Theorems for Real Functions of One Variable[1]

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Summary. Three basic theorems in differential calculus of one variable functions are presented: Rolle Theorem, Lagrange Theorem and Cauchy Theorem. There are also direct conclusions.

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WWW: http://mizar.org/JFM/Vol2/rolle.html

The articles [9], [11], [1], [10], [2], [12], [3], [4], [5], [8], [6], and [7] provide the notation and terminology for this paper.

We use the following convention: g, r, s, p, t, x, x_0, x_1 denote real numbers and f, f_1, f_2 denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

We now state a number of propositions:

(1) Let given p, g. Suppose $p < g$. Let given f. Suppose f is continuous on $[p,g]$ and $f(p) = f(g)$ and f is differentiable on $]p,g[$. Then there exists x_0 such that $x_0 \in]p,g[$ and $f'(x_0) = 0$.

(2) Let given x, t. Suppose $0 < t$. Let given f. Suppose f is continuous on $[x,x+t]$ and $f(x) = f(x+t)$ and f is differentiable on $]x,x+t[$. Then there exists s such that $0 < s$ and $s < 1$ and $f'(x+s\cdot t) = 0$.

(3) Let given p, g. Suppose $p < g$. Let given f. Suppose f is continuous on $[p,g]$ and differentiable on $]p,g[$. Then there exists x_0 such that $x_0 \in]p,g[$ and $f'(x_0) = \frac{f(g)-f(p)}{g-p}$.

(4) Let given x, t. Suppose $0 < t$. Let given f. Suppose f is continuous on $[x,x+t]$ and differentiable on $]x,x+t[$. Then there exists s such that $0 < s$ and $s < 1$ and $f(x+t) = f(x)+t\cdot f'(x+s\cdot t)$.

(5) Let given p, g. Suppose $p < g$. Let given f_1, f_2. Suppose f_1 is continuous on $[p,g]$ and differentiable on $]p,g[$ and f_2 is continuous on $[p,g]$ and differentiable on $]p,g[$. Then there exists x_0 such that $x_0 \in]p,g[$ and $(f_1(g)-f_1(p))\cdot {f_2}'(x_0) = (f_2(g)-f_2(p))\cdot {f_1}'(x_0)$.

(6) Let given x, t. Suppose $0 < t$. Let given f_1, f_2. Suppose that

(i) f_1 is continuous on $[x,x+t]$ and differentiable on $]x,x+t[$,

(ii) f_2 is continuous on $[x,x+t]$ and differentiable on $]x,x+t[$, and

(iii) for every x_1 such that $x_1 \in]x,x+t[$ holds ${f_2}'(x_1) \neq 0$.

Then there exists s such that $0 < s$ and $s < 1$ and $\frac{f_1(x+t)-f_1(x)}{f_2(x+t)-f_2(x)} = \frac{{f_1}'(x+s\cdot t)}{{f_2}'(x+s\cdot t)}$.

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(7) Let given p, g. Suppose $p < g$. Let given f. Suppose f is differentiable on $]p,g[$ and for every x such that $x \in]p,g[$ holds $f'(x) = 0$. Then f is a constant on $]p,g[$.

(8) Let given p, g. Suppose $p < g$. Let given f_1, f_2. Suppose f_1 is differentiable on $]p,g[$ and f_2 is differentiable on $]p,g[$ and for every x such that $x \in]p,g[$ holds ${f_1}'(x) = {f_2}'(x)$. Then $f_1 - f_2$ is a constant on $]p,g[$ and there exists r such that for every x such that $x \in]p,g[$ holds $f_1(x) = f_2(x) + r$.

(9) Let given p, g. Suppose $p < g$. Let given f. Suppose f is differentiable on $]p,g[$ and for every x such that $x \in]p,g[$ holds $0 < f'(x)$. Then f is increasing on $]p,g[$.

(10) Let given p, g. Suppose $p < g$. Let given f. Suppose f is differentiable on $]p,g[$ and for every x such that $x \in]p,g[$ holds $f'(x) < 0$. Then f is decreasing on $]p,g[$.

(11) Let given p, g. Suppose $p < g$. Let given f. Suppose f is differentiable on $]p,g[$ and for every x such that $x \in]p,g[$ holds $0 \leq f'(x)$. Then f is non-decreasing on $]p,g[$.

(12) Let given p, g. Suppose $p < g$. Let given f. Suppose f is differentiable on $]p,g[$ and for every x such that $x \in]p,g[$ holds $f'(x) \leq 0$. Then f is non increasing on $]p,g[$.

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