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Quadratic Inequalities

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Summary. Consider a quadratic trinomial of the form $P(x) = ax^2 + bx + c$, where $a \neq 0$. The determinant of the equation $P(x) = 0$ is of the form $\Delta(a,b,c) = b^2 - 4ac$. We prove several quadratic inequalities when $\Delta(a,b,c) < 0$, $\Delta(a,b,c) = 0$ and $\Delta(a,b,c) > 0$.

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The articles [1] and [2] provide the notation and terminology for this paper.

In this paper x, a, b, c denote real numbers.

Let us consider a, b, c. The functor $\Delta(a,b,c)$ is defined as follows:

(Def. 1) $\Delta(a,b,c) = b^2 - 4 \cdot a \cdot c$.

Let us consider a, b, c. Note that $\Delta(a,b,c)$ is real.

Let a, b, c be real numbers. Then $\Delta(a,b,c)$ is a real number.

We now state a number of propositions:

(1) If $a \neq 0$, then $a \cdot x^2 + b \cdot x + c = a \cdot (x + \frac{b}{2 \cdot a})^2 - \frac{\Delta(a,b,c)}{4 \cdot a}$.

(2) If $a > 0$ and $\Delta(a,b,c) \leq 0$, then $a \cdot x^2 + b \cdot x + c \geq 0$.

(3) If $a > 0$ and $\Delta(a,b,c) < 0$, then $a \cdot x^2 + b \cdot x + c > 0$.

(4) If $a < 0$ and $\Delta(a,b,c) \leq 0$, then $a \cdot x^2 + b \cdot x + c \leq 0$.

(5) If $a < 0$ and $\Delta(a,b,c) < 0$, then $a \cdot x^2 + b \cdot x + c < 0$.

(6) If $a > 0$ and $a \cdot x^2 + b \cdot x + c \geq 0$, then $(2 \cdot a \cdot x + b)^2 - \Delta(a,b,c) \geq 0$.

(7) If $a > 0$ and $a \cdot x^2 + b \cdot x + c > 0$, then $(2 \cdot a \cdot x + b)^2 - \Delta(a,b,c) > 0$.

(8) If $a < 0$ and $a \cdot x^2 + b \cdot x + c \leq 0$, then $(2 \cdot a \cdot x + b)^2 - \Delta(a,b,c) \geq 0$.

(9) If $a < 0$ and $a \cdot x^2 + b \cdot x + c < 0$, then $(2 \cdot a \cdot x + b)^2 - \Delta(a,b,c) > 0$.

(10) If for every x holds $a \cdot x^2 + b \cdot x + c \geq 0$ and $a > 0$, then $\Delta(a,b,c) \leq 0$.

(11) If for every x holds $a \cdot x^2 + b \cdot x + c \leq 0$ and $a < 0$, then $\Delta(a,b,c) \leq 0$.

(12) If for every x holds $a \cdot x^2 + b \cdot x + c > 0$ and $a > 0$, then $\Delta(a,b,c) < 0$.

(13) If for every x holds $a \cdot x^2 + b \cdot x + c < 0$ and $a < 0$, then $\Delta(a,b,c) < 0$.

(14) If $a \neq 0$ and $a \cdot x^2 + b \cdot x + c = 0$, then $(2 \cdot a \cdot x + b)^2 - \Delta(a,b,c) = 0$.

(15) If $a \neq 0$ and $\Delta(a,b,c) \geq 0$ and $a \cdot x^{2} + b \cdot x + c = 0$, then $x = \frac{-b-\sqrt{\Delta(a,b,c)}}{2 \cdot a}$ or $x = \frac{-b+\sqrt{\Delta(a,b,c)}}{2 \cdot a}$.

(16) If $a \neq 0$ and $\Delta(a,b,c) \geq 0$, then $a \cdot x^{2} + b \cdot x + c = a \cdot (x - \frac{-b-\sqrt{\Delta(a,b,c)}}{2 \cdot a}) \cdot (x - \frac{-b+\sqrt{\Delta(a,b,c)}}{2 \cdot a})$.

(17) If $a < 0$ and $\Delta(a,b,c) > 0$, then $\frac{-b+\sqrt{\Delta(a,b,c)}}{2 \cdot a} < \frac{-b-\sqrt{\Delta(a,b,c)}}{2 \cdot a}$.

(18) If $a < 0$ and $\Delta(a,b,c) > 0$, then $a \cdot x^{2} + b \cdot x + c > 0$ iff $\frac{-b+\sqrt{\Delta(a,b,c)}}{2 \cdot a} < x$ and $x < \frac{-b-\sqrt{\Delta(a,b,c)}}{2 \cdot a}$.

(19) If $a < 0$ and $\Delta(a,b,c) > 0$, then $a \cdot x^{2} + b \cdot x + c < 0$ iff $x < \frac{-b+\sqrt{\Delta(a,b,c)}}{2 \cdot a}$ or $x > \frac{-b-\sqrt{\Delta(a,b,c)}}{2 \cdot a}$.

(22)[1] If $a \neq 0$ and $\Delta(a,b,c) = 0$ and $a \cdot x^{2} + b \cdot x + c = 0$, then $x = -\frac{b}{2 \cdot a}$.

(23) If $a > 0$ and $(2 \cdot a \cdot x + b)^{2} - \Delta(a,b,c) > 0$, then $a \cdot x^{2} + b \cdot x + c > 0$.

(24) If $a > 0$ and $\Delta(a,b,c) = 0$, then $a \cdot x^{2} + b \cdot x + c > 0$ iff $x \neq -\frac{b}{2 \cdot a}$.

(25) If $a < 0$ and $(2 \cdot a \cdot x + b)^{2} - \Delta(a,b,c) > 0$, then $a \cdot x^{2} + b \cdot x + c < 0$.

(26) If $a < 0$ and $\Delta(a,b,c) = 0$, then $a \cdot x^{2} + b \cdot x + c < 0$ iff $x \neq -\frac{b}{2 \cdot a}$.

(27) If $a > 0$ and $\Delta(a,b,c) > 0$, then $\frac{-b+\sqrt{\Delta(a,b,c)}}{2 \cdot a} > \frac{-b-\sqrt{\Delta(a,b,c)}}{2 \cdot a}$.

(28) If $a > 0$ and $\Delta(a,b,c) > 0$, then $a \cdot x^{2} + b \cdot x + c < 0$ iff $\frac{-b-\sqrt{\Delta(a,b,c)}}{2 \cdot a} < x$ and $x < \frac{-b+\sqrt{\Delta(a,b,c)}}{2 \cdot a}$.

(29) If $a > 0$ and $\Delta(a,b,c) > 0$, then $a \cdot x^{2} + b \cdot x + c > 0$ iff $x < \frac{-b-\sqrt{\Delta(a,b,c)}}{2 \cdot a}$ or $x > \frac{-b+\sqrt{\Delta(a,b,c)}}{2 \cdot a}$.

REFERENCES

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[2] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers operations: min, max, square, and square root. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/square_1.html`.

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[1] The propositions (20) and (21) have been removed.