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Incidence Projective Space (a reduction theorem in a plane)[1]

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Summary. The article begins with basic facts concerning arbitrary projective spaces. Further we are concerned with Fano projective spaces (we prove it has rank at least four). Finally we restrict ourselves to Desarguesian planes; we define the notion of perspectivity and we prove the reduction theorem for projectivities with concurrent axes.

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The articles [3], [5], [6], [7], [4], [2], and [1] provide the notation and terminology for this paper.

We adopt the following rules: I_1 denotes a projective space defined in terms of incidence, a, b, c, d, p, q, o, r, s denote points of I_1, and A, B, C, P, Q denote lines of I_1.

Next we state a number of propositions:

(1) There exists a such that a does not lie on A.

(2) There exists A such that a does not lie on A.

(3) If $A \neq B$, then there exist a, b such that a lies on A and a does not lie on B and b lies on B and b does not lie on A.

(4) If $a \neq b$, then there exist A, B such that a lies on A and a does not lie on B and b lies on B and b does not lie on A.

(5) There exist A, B, C such that a lies on A and a lies on B and a lies on C and $A \neq B$ and $B \neq C$ and $C \neq A$.

(6) There exists a such that a does not lie on A and a does not lie on B.

(7) There exists a such that a lies on A.

(8) There exists c such that c lies on A and $c \neq a$ and $c \neq b$.

(9) There exists A such that a does not lie on A and b does not lie on A.

(12)[1] Suppose that o lies on A and o lies on B and $A \neq B$ and a lies on A and $o \neq a$ and b lies on B and c lies on B and $b \neq c$ and a lies on P and b lies on P and a lies on Q and c lies on Q. Then $P \neq Q$.

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[1] The propositions (10) and (11) have been removed.

(13) Suppose a, b, c lie on A. Then a, c, b lie on A and b, a, c lie on A and b, c, a lie on A and c, a, b lie on A and c, b, a lie on A.

(14) Let I_1 be a Desarguesian projective space defined in terms of incidence, o, b_1, a_1, b_2, a_2, b_3, a_3, r, s, t be points of I_1, and C_1, C_2, C_3, A_1, A_2, A_3, B_1, B_2, B_3 be lines of I_1. Suppose that o, b_1, a_1 lie on C_1 and o, a_2, b_2 lie on C_2 and o, a_3, b_3 lie on C_3 and a_3, a_2, t lie on A_1 and a_3, r, a_1 lie on A_2 and a_2, s, a_1 lie on A_3 and t, b_2, b_3 lie on B_1 and b_1, r, b_3 lie on B_2 and b_1, s, b_2 lie on B_3 and C_1, C_2, C_3 are mutually different and $o \neq a_3$ and $o \neq b_1$ and $o \neq b_2$ and $a_2 \neq b_2$. Then there exists a line O of I_1 such that r, s, t lie on O.

(15) Given A, a, b, c, d such that a lies on A and b lies on A and c lies on A and d lies on A and a, b, c, d are mutually different. Let given B. Then there exist p, q, r, s such that p lies on B and q lies on B and r lies on B and s lies on B and p, q, r, s are mutually different.

We use the following convention: I_1 is a Fanoian projective space defined in terms of incidence, a, b, c, d, p, q, r, s are points of I_1, and A, B, C, D, L, Q, R, S are lines of I_1.

Next we state four propositions:

(16) There exist p, q, r, s, a, b, c, A, B, C, Q, L, R, S, D such that

q does not lie on L and r does not lie on L and p does not lie on Q and s does not lie on Q and p does not lie on R and r does not lie on R and q does not lie on S and s does not lie on S and a, p, s lie on L and a, q, r lie on Q and b, q, s lie on R and b, p, r lie on S and c, p, q lie on A and c, r, s lie on B and a, b lie on C and c does not lie on C.

(17) There exist a, A, B, C, D such that a lies on A and a lies on B and a lies on C and a lies on D and A, B, C, D are mutually different.

(18) There exist a, b, c, d, A such that a lies on A and b lies on A and c lies on A and d lies on A and a, b, c, d are mutually different.

(19) There exist p, q, r, s such that p lies on B and q lies on B and r lies on B and s lies on B and p, q, r, s are mutually different.

We adopt the following convention: I_1 is a Desarguesian 2-dimensional projective space defined in terms of incidence, c, p, q, x, y are points of I_1, and K, L, R, X are lines of I_1.

Let us consider I_1, K, L, p. Let us assume that p does not lie on K and p does not lie on L. The functor $\pi_p(K \to L)$ yields a partial function from the points of I_1 to the points of I_1 and is defined by the conditions (Def. 1).

(Def. 1)(i) $\mathrm{dom}\,\pi_p(K \to L) \subseteq$ the points of I_1,

(ii) for every x holds $x \in \mathrm{dom}\,\pi_p(K \to L)$ iff x lies on K, and

(iii) for all x, y such that x lies on K and y lies on L holds $\pi_p(K \to L)(x) = y$ iff there exists X such that p lies on X and x lies on X and y lies on X.

We now state several propositions:

(21)[2] If p does not lie on K, then for every x such that x lies on K holds $\pi_p(K \to K)(x) = x$.

(22) If p does not lie on K and p does not lie on L and x lies on K, then $\pi_p(K \to L)(x)$ is a point of I_1.

(23) If p does not lie on K and p does not lie on L and x lies on K and $y = \pi_p(K \to L)(x)$, then y lies on L.

(24) If p does not lie on K and p does not lie on L and $y \in \mathrm{rng}\,\pi_p(K \to L)$, then y lies on L.

(25) Suppose p does not lie on K and p does not lie on L and q does not lie on L and q does not lie on R. Then $\mathrm{dom}(\pi_q(L \to R) \cdot \pi_p(K \to L)) = \mathrm{dom}\,\pi_p(K \to L)$ and $\mathrm{rng}(\pi_q(L \to R) \cdot \pi_p(K \to L)) = \mathrm{rng}\,\pi_q(L \to R)$.

[2] The proposition (20) has been removed.

(26) Let a_1, b_1, a_2, b_2 be points of I_1. Suppose p does not lie on K and p does not lie on L and a_1 lies on K and b_1 lies on K and $\pi_p(K \to L)(a_1) = a_2$ and $\pi_p(K \to L)(b_1) = b_2$ and $a_2 = b_2$. Then $a_1 = b_1$.

(27) If p does not lie on K and p does not lie on L and x lies on K and x lies on L, then $\pi_p(K \to L)(x) = x$.

(28) Suppose that p does not lie on K and p does not lie on L and q does not lie on L and q does not lie on R and c lies on K and c lies on L and c lies on R and $K \neq R$. Then there exists a point o of I_1 such that o does not lie on K and o does not lie on R and $\pi_q(L \to R) \cdot \pi_p(K \to L) = \pi_o(K \to R)$.

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