JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

Desargues Theorem In Projective 3-Space

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Summary. Proof of the Desargues theorem in Fanoian projective at least 3-dimensional space.

MML Identifier: PROJDES1.

WWW: http://mizar.org/JFM/Vol2/projdes1.html

The articles [2] and [1] provide the notation and terminology for this paper.

We adopt the following convention: F_1 denotes an up 3-dimensional projective space defined in terms of collinearity and a, a', b, b', c, c', d, d', o, p, q, r, s, t, u, x denote elements of F_1.

The following propositions are true:

(1) Suppose a, b and c are collinear. Then

(i) b, c and a are collinear,

(ii) c, a and b are collinear,

(iii) b, a and c are collinear,

(iv) a, c and b are collinear, and

(v) c, b and a are collinear.

(5)[1] There exist q, r such that p, q and r are not collinear.

(6) If a, b and c are not collinear and a, b and b' are collinear and $a \neq b'$, then a, b' and c are not collinear.

(7) If a, b and c are not collinear and a, b and d are collinear and a, c and d are collinear, then $a = d$.

(8) Suppose that o, a and d are not collinear and o, d and d' are collinear and a, d and s are collinear and $d \neq d'$ and a', d' and s are collinear and o, a and a' are collinear and $o \neq a'$. Then $s \neq d$.

Let us consider F_1, a, b, c, d. We say that a, b, c, d are coplanar if and only if:

(Def. 1) There exists an element x of F_1 such that a, b and x are collinear and c, d and x are collinear.

One can prove the following propositions:

[1] The propositions (2)–(4) have been removed.

(10)[2] Suppose that

(i) a, b and c are collinear, or

(ii) b, c and d are collinear, or

(iii) c, d and a are collinear, or

(iv) d, a and b are collinear.

Then a, b, c, d are coplanar.

(11) Suppose a, b, c, d are coplanar. Then b, c, d, a are coplanar and c, d, a, b are coplanar and d, a, b, c are coplanar and b, a, c, d are coplanar and c, b, d, a are coplanar and d, c, a, b are coplanar and a, d, b, c are coplanar and a, c, d, b are coplanar and b, d, a, c are coplanar and c, a, b, d are coplanar and d, b, c, a are coplanar and c, a, d, b are coplanar and d, b, a, c are coplanar and a, c, b, d are coplanar and b, d, c, a are coplanar and a, b, d, c are coplanar and a, d, c, b are coplanar and b, c, a, d are coplanar and b, a, d, c are coplanar and c, b, a, d are coplanar and c, d, b, a are coplanar and d, a, c, b are coplanar and d, c, b, a are coplanar.

(12) Suppose that

(i) a, b and c are not collinear,

(ii) a, b, c, p are coplanar,

(iii) a, b, c, q are coplanar,

(iv) a, b, c, r are coplanar, and

(v) a, b, c, s are coplanar.

Then p, q, r, s are coplanar.

(13) Suppose that

(i) p, q and r are not collinear,

(ii) a, b, c, p are coplanar,

(iii) a, b, c, r are coplanar,

(iv) a, b, c, q are coplanar, and

(v) p, q, r, s are coplanar.

Then a, b, c, s are coplanar.

(14) Suppose $p \neq q$ and p, q and r are collinear and a, b, c, p are coplanar and a, b, c, q are coplanar. Then a, b, c, r are coplanar.

(15) Suppose that

(i) a, b and c are not collinear,

(ii) a, b, c, p are coplanar,

(iii) a, b, c, q are coplanar,

(iv) a, b, c, r are coplanar, and

(v) a, b, c, s are coplanar.

Then there exists x such that p, q and x are collinear and r, s and x are collinear.

(16) There exist a, b, c, d such that a, b, c, d are not coplanar.

(17) If p, q and r are not collinear, then there exists s such that p, q, r, s are not coplanar.

(18) If $a = b$ or $a = c$ or $b = c$ or $a = d$ or $b = d$ or $d = c$, then a, b, c, d are coplanar.

(19) If a, b, c, o are not coplanar and o, a and a' are collinear and $a \neq a'$, then a, b, c, a' are not coplanar.

[2] The proposition (9) has been removed.

(20) Suppose that a, b and c are not collinear and a', b' and c' are not collinear and a, b, c, p are coplanar and a, b, c, q are coplanar and a, b, c, r are coplanar and a', b', c', p are coplanar and a', b', c', q are coplanar and a', b', c', r are coplanar and a, b, c, a' are not coplanar. Then p, q and r are collinear.

(21) Suppose that $a \neq a'$ and o, a and a' are collinear and a, b, c, o are not coplanar and a', b' and c' are not collinear and a, b and p are collinear and a', b' and p are collinear and b, c and q are collinear and b', c' and q are collinear and a, c and r are collinear and a', c' and r are collinear. Then p, q and r are collinear.

(22) Suppose a, b, c, d are not coplanar and a, b, c, o are coplanar and a, b and o are not collinear. Then a, b, d, o are not coplanar.

(23) Suppose that a, b, c, o are not coplanar and o, a and a' are collinear and o, b and b' are collinear and o, c and c' are collinear and $o \neq a'$ and $o \neq b'$ and $o \neq c'$. Then a', b' and c' are not collinear and a', b', c', o are not coplanar.

(24) Suppose that a, b, c, o are coplanar and a, b, c, d are not coplanar and a, b, d, o are not coplanar and b, c, d, o are not coplanar and a, c, d, o are not coplanar and o, d and d' are collinear and o, a and a' are collinear and o, b and b' are collinear and o, c and c' are collinear and a, d and s are collinear and a', d' and s are collinear and b, d and t are collinear and b', d' and t are collinear and c, d and u are collinear and $o \neq a'$ and $o \neq d'$ and $d \neq d'$ and $o \neq b'$. Then s, t and u are not collinear.

Let us consider F_1, o, a, b, c. We say that o, a, b, and c constitute a quadrangle if and only if the conditions (Def. 2) are satisfied.

(Def. 2)(i) a, b and c are not collinear,

(ii) o, a and b are not collinear,

(iii) o, b and c are not collinear, and

(iv) o, c and a are not collinear.

Next we state two propositions:

(26)[3] Suppose that o, a and b are not collinear and o, b and c are not collinear and o, a and c are not collinear and o, a and a' are collinear and o, b and b' are collinear and o, c and c' are collinear and a, b and p are collinear and a', b' and p are collinear and $a \neq a'$ and b, c and r are collinear and b', c' and r are collinear and a, c and q are collinear and $b \neq b'$ and a', c' and q are collinear and $o \neq a'$ and $o \neq b'$ and $o \neq c'$. Then r, q and p are collinear.

(27) Every up 3-dimensional projective space defined in terms of collinearity is Desarguesian.

Let us note that every up 3-dimensional projective space defined in terms of collinearity is Desarguesian.

References

[1] Wojciech Leończuk and Krzysztof Prażmowski. Projective spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/anproj_2.html.

[3] The proposition (25) has been removed.

[2] Wojciech Skaba. The collinearity structure. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/collsp.html.

Received August 13, 1990

Published January 2, 2004