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Calculus of Propositions

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Summary. Continues the analysis of classical language of first order (see [6], [1], [3], [4], [2]). Three connectives: truth, negation and conjuction are primary (see [6]). The others (alternative, implication and equivalence) are defined with respect to them (see [1]). We prove some important tautologies of the calculus of propositions. Most of them are given as the axioms of classical logical calculus (see [5]). In the last part of our article we give some basic rules of inference.

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The articles [7], [3], and [4] provide the notation and terminology for this paper.

In this paper p, q, r, s are elements of CQC-WFF.

The following propositions are true:

(1) $\neg(p \wedge \neg p) \in \mathrm{Taut}$.

(2) $p \vee \neg p \in \mathrm{Taut}$.

(3) $p \Rightarrow p \vee q \in \mathrm{Taut}$.

(4) $q \Rightarrow p \vee q \in \mathrm{Taut}$.

(5) $p \vee q \Rightarrow (\neg p \Rightarrow q) \in \mathrm{Taut}$.

(6) $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q \in \mathrm{Taut}$.

(7) $\neg p \wedge \neg q \Rightarrow \neg(p \vee q) \in \mathrm{Taut}$.

(8) $p \vee q \Rightarrow q \vee p \in \mathrm{Taut}$.

(9) $\neg p \vee p \in \mathrm{Taut}$.

(10) $\neg(p \vee q) \Rightarrow \neg p \in \mathrm{Taut}$.

(11) $p \vee p \Rightarrow p \in \mathrm{Taut}$.

(12) $p \Rightarrow p \vee p \in \mathrm{Taut}$.

(13) $p \wedge \neg p \Rightarrow q \in \mathrm{Taut}$.

(14) $(p \Rightarrow q) \Rightarrow \neg p \vee q \in \mathrm{Taut}$.

(15) $p \wedge q \Rightarrow \neg(p \Rightarrow \neg q) \in \mathrm{Taut}$.

(16) $\neg(p \Rightarrow \neg q) \Rightarrow p \wedge q \in \mathrm{Taut}$.

(17) $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q \in \mathrm{Taut}$.

(18) $\neg p \vee \neg q \Rightarrow \neg(p \wedge q) \in \mathrm{Taut}$.

(19) $p \wedge q \Rightarrow p \in \mathrm{Taut}$.

(20) $p \wedge q \Rightarrow p \vee q \in \mathrm{Taut}$.

(21) $p \wedge q \Rightarrow q \in \mathrm{Taut}$.

(22) $p \Rightarrow p \wedge p \in \mathrm{Taut}$.

(23) $(p \Leftrightarrow q) \Rightarrow (p \Rightarrow q) \in \mathrm{Taut}$.

(24) $(p \Leftrightarrow q) \Rightarrow (q \Rightarrow p) \in \mathrm{Taut}$.

(25) $p \vee q \vee r \Rightarrow p \vee (q \vee r) \in \mathrm{Taut}$.

(26) $p \wedge q \wedge r \Rightarrow p \wedge (q \wedge r) \in \mathrm{Taut}$.

(27) $p \vee (q \vee r) \Rightarrow p \vee q \vee r \in \mathrm{Taut}$.

(28) $p \Rightarrow (q \Rightarrow p \wedge q) \in \mathrm{Taut}$.

(29) $(p \Rightarrow q) \Rightarrow ((q \Rightarrow p) \Rightarrow (p \Leftrightarrow q)) \in \mathrm{Taut}$.

(30) $p \vee q \Leftrightarrow q \vee p \in \mathrm{Taut}$.

(31) $(p \wedge q \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r)) \in \mathrm{Taut}$.

(32) $(p \Rightarrow (q \Rightarrow r)) \Rightarrow (p \wedge q \Rightarrow r) \in \mathrm{Taut}$.

(33) $(r \Rightarrow p) \Rightarrow ((r \Rightarrow q) \Rightarrow (r \Rightarrow p \wedge q)) \in \mathrm{Taut}$.

(34) $(p \vee q \Rightarrow r) \Rightarrow (p \Rightarrow r) \vee (q \Rightarrow r) \in \mathrm{Taut}$.

(35) $(p \Rightarrow r) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \vee q \Rightarrow r)) \in \mathrm{Taut}$.

(36) $(p \Rightarrow r) \wedge (q \Rightarrow r) \Rightarrow (p \vee q \Rightarrow r) \in \mathrm{Taut}$.

(37) $(p \Rightarrow q \wedge \neg q) \Rightarrow \neg p \in \mathrm{Taut}$.

(38) $(p \vee q) \wedge (p \vee r) \Rightarrow p \vee q \wedge r \in \mathrm{Taut}$.

(39) $p \wedge (q \vee r) \Rightarrow p \wedge q \vee p \wedge r \in \mathrm{Taut}$.

(40) $(p \vee r) \wedge (q \vee r) \Rightarrow p \wedge q \vee r \in \mathrm{Taut}$.

(41) $(p \vee q) \wedge r \Rightarrow p \wedge r \vee q \wedge r \in \mathrm{Taut}$.

(42) If $p \in \mathrm{Taut}$, then $p \vee q \in \mathrm{Taut}$.

(43) If $q \in \mathrm{Taut}$, then $p \vee q \in \mathrm{Taut}$.

(44) If $p \wedge q \in \mathrm{Taut}$, then $p \in \mathrm{Taut}$.

(45) If $p \wedge q \in \mathrm{Taut}$, then $q \in \mathrm{Taut}$.

(46) If $p \wedge q \in \mathrm{Taut}$, then $p \vee q \in \mathrm{Taut}$.

(47) If $p \in \mathrm{Taut}$ and $q \in \mathrm{Taut}$, then $p \wedge q \in \mathrm{Taut}$.

(48) If $p \Rightarrow q \in \mathrm{Taut}$, then $p \vee r \Rightarrow q \vee r \in \mathrm{Taut}$.

(49) If $p \Rightarrow q \in \mathrm{Taut}$, then $r \vee p \Rightarrow r \vee q \in \mathrm{Taut}$.

(50) If $p \Rightarrow q \in \mathrm{Taut}$, then $r \wedge p \Rightarrow r \wedge q \in \mathrm{Taut}$.

(51) If $p \Rightarrow q \in \mathrm{Taut}$, then $p \wedge r \Rightarrow q \wedge r \in \mathrm{Taut}$.

(52) If $r \Rightarrow p \in \mathrm{Taut}$ and $r \Rightarrow q \in \mathrm{Taut}$, then $r \Rightarrow p \wedge q \in \mathrm{Taut}$.

(53) If $p \Rightarrow r \in \mathrm{Taut}$ and $q \Rightarrow r \in \mathrm{Taut}$, then $p \vee q \Rightarrow r \in \mathrm{Taut}$.

(54) If $p \vee q \in \mathrm{Taut}$ and $\neg p \in \mathrm{Taut}$, then $q \in \mathrm{Taut}$.

(55) If $p \vee q \in \mathrm{Taut}$ and $\neg q \in \mathrm{Taut}$, then $p \in \mathrm{Taut}$.

(56) If $p \Rightarrow q \in \mathrm{Taut}$ and $r \Rightarrow s \in \mathrm{Taut}$, then $p \wedge r \Rightarrow q \wedge s \in \mathrm{Taut}$.

(57) If $p \Rightarrow q \in \mathrm{Taut}$ and $r \Rightarrow s \in \mathrm{Taut}$, then $p \vee r \Rightarrow q \vee s \in \mathrm{Taut}$.

(58) If $p \wedge \neg q \Rightarrow \neg p \in \mathrm{Taut}$, then $p \Rightarrow q \in \mathrm{Taut}$.

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