

On Cosets in Segre's Product of Partial Linear Spaces

Adam Naumowicz
University of Białystok

Summary. This paper is a continuation of [15]. We prove that the family of cosets in the Segre's product of partial linear spaces remains invariant under automorphisms.

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The articles [19], [10], [23], [2], [24], [16], [7], [9], [8], [5], [13], [1], [4], [3], [17], [20], [21], [6], [11], [25], [14], [18], [12], [22], and [15] provide the notation and terminology for this paper.

1. PRELIMINARIES ON FINITE SEQUENCES

Let D be a set, let p be a finite sequence of elements of D , and let i, j be natural numbers. The functor $\text{Del}(p, i, j)$ yielding a finite sequence of elements of D is defined by:

(Def. 1) $\text{Del}(p, i, j) = (p \upharpoonright (i - 1)) \hat{\ } (p \downharpoonright j)$.

Next we state several propositions:

- (1) For every set D and for every finite sequence p of elements of D and for all natural numbers i, j holds $\text{rng Del}(p, i, j) \subseteq \text{rng } p$.
- (2) Let D be a set, p be a finite sequence of elements of D , and i, j be natural numbers. If $i \in \text{dom } p$ and $j \in \text{dom } p$, then $\text{len Del}(p, i, j) = ((\text{len } p - j) + i) - 1$.
- (3) Let D be a set, p be a finite sequence of elements of D , and i, j be natural numbers. If $i \in \text{dom } p$ and $j \in \text{dom } p$, then if $\text{len Del}(p, i, j) = 0$, then $i = 1$ and $j = \text{len } p$.
- (4) Let D be a set, p be a finite sequence of elements of D , and i, j, k be natural numbers. If $i \in \text{dom } p$ and $1 \leq k$ and $k \leq i - 1$, then $(\text{Del}(p, i, j))(k) = p(k)$.
- (5) For all finite sequences p, q and for every natural number k such that $\text{len } p + 1 \leq k$ holds $(p \hat{\ } q)(k) = q(k - \text{len } p)$.
- (6) Let D be a set, p be a finite sequence of elements of D , and i, j, k be natural numbers. Suppose $i \in \text{dom } p$ and $j \in \text{dom } p$ and $i \leq j$ and $i \leq k$ and $k \leq ((\text{len } p - j) + i) - 1$. Then $(\text{Del}(p, i, j))(k) = p((j - i) + k + 1)$.

The scheme *FinSeqOneToOne* deals with sets $\mathcal{A}, \mathcal{B}, \mathcal{C}$, a finite sequence \mathcal{D} of elements of \mathcal{C} , and a binary predicate \mathcal{P} , and states that:

There exists an one-to-one finite sequence g of elements of \mathcal{C} such that $\mathcal{A} = g(1)$ and $\mathcal{B} = g(\text{len } g)$ and $\text{rng } g \subseteq \text{rng } \mathcal{D}$ and for every natural number j such that $1 \leq j$ and $j < \text{len } g$ holds $\mathcal{P}[g(j), g(j + 1)]$

provided the parameters satisfy the following conditions:

- $\mathcal{A} = \mathcal{D}(1)$ and $\mathcal{B} = \mathcal{D}(\text{len } \mathcal{D})$, and
- For every natural number i and for all sets d_1, d_2 such that $1 \leq i$ and $i < \text{len } \mathcal{D}$ and $d_1 = \mathcal{D}(i)$ and $d_2 = \mathcal{D}(i+1)$ holds $\mathcal{P}[d_1, d_2]$.

2. SEGRE COSETS

We now state the proposition

- (7) Let I be a non empty set, A be a 1-sorted yielding many sorted set indexed by I , L be a many sorted subset indexed by the support of A , i be an element of I , and S be a subset of $A(i)$. Then $L + \cdot (i, S)$ is a many sorted subset indexed by the support of A .

Let I be a non empty set and let A be a non-Trivial-yielding TopStruct-yielding many sorted set indexed by I . A subset of Segre_Product A is called a Segre-Coset of A if it satisfies the condition (Def. 2).

- (Def. 2) There exists a Segre-like non trivial-yielding many sorted subset L indexed by the support of A such that it = $\prod L$ and $L(\text{index}(L)) = \Omega_{A(\text{index}(L))}$.

One can prove the following proposition

- (8) Let I be a non empty set, A be a non-Trivial-yielding TopStruct-yielding many sorted set indexed by I , and B_1, B_2 be Segre-Cosets of A . If $2 \subseteq \overline{B_1 \cap B_2}$, then $B_1 = B_2$.

Let S be a topological structure and let X, Y be subsets of S . We say that X and Y are joinable if and only if the condition (Def. 3) is satisfied.

- (Def. 3) There exists a finite sequence f of elements of $2^{\text{the carrier of } S}$ such that

- (i) $X = f(1)$,
- (ii) $Y = f(\text{len } f)$,
- (iii) for every subset W of S such that $W \in \text{rng } f$ holds W is closed under lines and strong, and
- (iv) for every natural number i such that $1 \leq i$ and $i < \text{len } f$ holds $2 \subseteq \overline{f(i) \cap f(i+1)}$.

The following propositions are true:

- (9) Let S be a topological structure and X, Y be subsets of S . Suppose X and Y are joinable. Then there exists an one-to-one finite sequence f of elements of $2^{\text{the carrier of } S}$ such that

- (i) $X = f(1)$,
- (ii) $Y = f(\text{len } f)$,
- (iii) for every subset W of S such that $W \in \text{rng } f$ holds W is closed under lines and strong, and
- (iv) for every natural number i such that $1 \leq i$ and $i < \text{len } f$ holds $2 \subseteq \overline{f(i) \cap f(i+1)}$.

- (10) Let S be a topological structure and X be a subset of S . If X is closed under lines and strong, then X and X are joinable.

- (11) Let I be a non empty set, A be a PLS-yielding many sorted set indexed by I , and X, Y be subsets of Segre_Product A . Suppose that

- (i) X is non trivial, closed under lines, and strong,
- (ii) Y is non trivial, closed under lines, and strong, and
- (iii) X and Y are joinable.

Let X_1, Y_1 be Segre-like non trivial-yielding many sorted subsets indexed by the support of A . Suppose $X = \prod X_1$ and $Y = \prod Y_1$. Then $\text{index}(X_1) = \text{index}(Y_1)$ and for every set i such that $i \neq \text{index}(X_1)$ holds $X_1(i) = Y_1(i)$.

3. COLLINEATIONS OF SEGRE PRODUCT

Next we state the proposition

- (12) Let S be a 1-sorted structure, T be a non empty 1-sorted structure, and f be a map from S into T . If f is bijective, then f^{-1} is bijective.

Let S, T be topological structures and let f be a map from S into T . We say that f is isomorphic if and only if:

- (Def. 4) f is bijective and open and f^{-1} is bijective and open.

Let S be a non empty topological structure. Observe that there exists a map from S into S which is isomorphic.

Let S be a non empty topological structure. A collineation of S is an isomorphic map from S into S .

Let S be a non empty non void topological structure, let f be a collineation of S , and let l be a block of S . Then $f \circ l$ is a block of S .

Let S be a non empty non void topological structure, let f be a collineation of S , and let l be a block of S . Then $f^{-1}(l)$ is a block of S .

Next we state a number of propositions:

- (13) For every non empty topological structure S and for every collineation f of S holds f^{-1} is a collineation of S .
- (14) Let S be a non empty topological structure, f be a collineation of S , and X be a subset of S . If X is non trivial, then $f \circ X$ is non trivial.
- (15) Let S be a non empty topological structure, f be a collineation of S , and X be a subset of S . If X is non trivial, then $f^{-1}(X)$ is non trivial.
- (16) Let S be a non empty non void topological structure, f be a collineation of S , and X be a subset of S . If X is strong, then $f \circ X$ is strong.
- (17) Let S be a non empty non void topological structure, f be a collineation of S , and X be a subset of S . If X is strong, then $f^{-1}(X)$ is strong.
- (18) Let S be a non empty non void topological structure, f be a collineation of S , and X be a subset of S . If X is closed under lines, then $f \circ X$ is closed under lines.
- (19) Let S be a non empty non void topological structure, f be a collineation of S , and X be a subset of S . If X is closed under lines, then $f^{-1}(X)$ is closed under lines.
- (20) Let S be a non empty non void topological structure, f be a collineation of S , and X, Y be subsets of S . Suppose X is non trivial and Y is non trivial and X and Y are joinable. Then $f \circ X$ and $f \circ Y$ are joinable.
- (21) Let S be a non empty non void topological structure, f be a collineation of S , and X, Y be subsets of S . Suppose X is non trivial and Y is non trivial and X and Y are joinable. Then $f^{-1}(X)$ and $f^{-1}(Y)$ are joinable.
- (22) Let I be a non empty set and A be a PLS-yielding many sorted set indexed by I . Suppose that for every element i of I holds $A(i)$ is strongly connected. Let W be a subset of $\text{Segre_Product}A$. Suppose W is non trivial, strong, and closed under lines. Then $\bigcup\{Y; Y \text{ ranges over subsets of } \text{Segre_Product}A : Y \text{ is non trivial, strong, and closed under lines } \wedge W \text{ and } Y \text{ are joinable}\}$ is a Segre-Coset of A .
- (23) Let I be a non empty set and A be a PLS-yielding many sorted set indexed by I . Suppose that for every element i of I holds $A(i)$ is strongly connected. Let B be a set. Then B is a Segre-Coset of A if and only if there exists a subset W of $\text{Segre_Product}A$ such that W is non trivial, strong, and closed under lines and $B = \bigcup\{Y; Y \text{ ranges over subsets of } \text{Segre_Product}A : Y \text{ is non trivial, strong, and closed under lines } \wedge W \text{ and } Y \text{ are joinable}\}$.

- (24) Let I be a non empty set and A be a PLS-yielding many sorted set indexed by I . Suppose that for every element i of I holds $A(i)$ is strongly connected. Let B be a Segre-Coset of A and f be a collineation of Segre_Product A . Then $f \circ B$ is a Segre-Coset of A .
- (25) Let I be a non empty set and A be a PLS-yielding many sorted set indexed by I . Suppose that for every element i of I holds $A(i)$ is strongly connected. Let B be a Segre-Coset of A and f be a collineation of Segre_Product A . Then $f^{-1}(B)$ is a Segre-Coset of A .

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