JOURNAL OF FORMALIZED MATHEMATICS
Volume 13, Released 2001, Published 2003
Inst. of Computer Science, Univ. of Białystok

Classes of Independent Partitions

Andrzej Trybulec
University of Białystok

Summary. The paper includes proofs of few theorems proved earlier by Shunichi Kobayashi in many different settings.

MML Identifier: PARTIT_2.

WWW: http://mizar.org/JFM/Vol13/partit_2.html

The articles [10], [4], [12], [8], [16], [15], [13], [17], [1], [14], [3], [2], [11], [9], [6], [5], and [7] provide the notation and terminology for this paper.

1. PRELIMINARIES

Let X, Y be sets and let R, S be relations between X and Y. Let us observe that $R \subseteq S$ if and only if:

(Def. 1) For every element x of X and for every element y of Y such that $\langle x, y\rangle \in R$ holds $\langle x, y\rangle \in S$.

For simplicity, we use the following convention: Y denotes a non empty set, a denotes an element of $\mathit{Boolean}^Y$, G denotes a subset of PARTITIONS(Y), and P, Q denote partitions of Y.

Let Y be a non empty set and let G be a non empty subset of PARTITIONS(Y). We see that the element of G is a partition of Y.

We now state a number of propositions:

(1) $\bigwedge \emptyset_{\mathrm{PARTITIONS}(Y)} = O(Y)$.

(2) For all equivalence relations R, S of Y holds $R \cup S \subseteq R \cdot S$.

(3) For every binary relation R on Y holds $R \subseteq \nabla_Y$.

(4) For every equivalence relation R of Y holds $\nabla_Y \cdot R = \nabla_Y$ and $R \cdot \nabla_Y = \nabla_Y$.

(5) For every partition P of Y and for all elements x, y of Y holds $\langle x, y\rangle \in \equiv_P$ iff $x \in \mathrm{EqClass}(y, P)$.

(6) Let P, Q, R be partitions of Y. Suppose $\equiv_R = \equiv_P \cdot \equiv_Q$. Let x, y be elements of Y. Then $x \in \mathrm{EqClass}(y, R)$ if and only if there exists an element z of Y such that $x \in \mathrm{EqClass}(z, P)$ and $z \in \mathrm{EqClass}(y, Q)$.

(7) Let R, S be binary relations and Y be a set. If R is reflexive in Y and S is reflexive in Y, then $R \cdot S$ is reflexive in Y.

(8) For every binary relation R and for every set Y such that R is reflexive in Y holds $Y \subseteq \mathrm{field}\, R$.

(9) For every set Y and for every binary relation R on Y such that R is reflexive in Y holds $Y = \mathrm{field}\, R$.

(10) For all equivalence relations R, S of Y such that $R \cdot S = S \cdot R$ holds $R \cdot S$ is an equivalence relation of Y.

2. Boolean-Valued Functions

The following three propositions are true:

(11) For all elements a, b of $Boolean^Y$ such that $a \Subset b$ holds $\neg b \Subset \neg a$.

(13)[1] Let a, b be elements of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P be a partition of Y. If $a \Subset b$, then $\forall_{a,P}G \Subset \forall_{b,P}G$.

(15)[2] Let a, b be elements of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P be a partition of Y. If $a \Subset b$, then $\exists_{a,P}G \Subset \exists_{b,P}G$.

3. Independent Classes of Partitions

One can prove the following propositions:

(16) If G is independent, then for all subsets P, Q of PARTITIONS(Y) such that $P \subseteq G$ and $Q \subseteq G$ holds $\equiv_{\bigwedge P} \cdot \equiv_{\bigwedge Q} = \equiv_{\bigwedge Q} \cdot \equiv_{\bigwedge P}$.

(17) If G is independent, then $\forall_{\forall_{a,P}G,Q}G = \forall_{\forall_{a,Q}G,P}G$.

(18) If G is independent, then $\exists_{\exists_{a,P}G,Q}G = \exists_{\exists_{a,Q}G,P}G$.

(19) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and P, Q be partitions of Y. If G is independent, then $\exists_{\forall_{a,P}G,Q}G \Subset \forall_{\exists_{a,Q}G,P}G$.

References

[1] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[2] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[3] Czesław Byliński. Partial functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/partfun1.html.

[4] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[5] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/bvfunc_1.html.

[6] Shunichi Kobayashi and Kui Jia. A theory of partitions. Part I. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/partit1.html.

[7] Shunichi Kobayashi and Yatsuka Nakamura. A theory of Boolean valued functions and quantifiers with respect to partitions. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/bvfunc_2.html.

[8] Beata Padlewska. Families of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/setfam_1.html.

[9] Konrad Raczkowski and Paweł Sadowski. Equivalence relations and classes of abstraction. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/eqrel_1.html.

[10] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[11] Andrzej Trybulec. Function domains and Frænkel operator. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/fraenkel.html.

[12] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[13] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

[14] Edmund Woronowicz. Relations defined on sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relset_1.html.

[15] Edmund Woronowicz. Interpretation and satisfiability in the first order logic. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/valuat_1.html.

[1] The proposition (12) has been removed.
[2] The proposition (14) has been removed.

[16] Edmund Woronowicz. Many-argument relations. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/margrel1.html.

[17] Edmund Woronowicz and Anna Zalewska. Properties of binary relations. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_2.html.

Received February 14, 2001

Published January 2, 2004