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Elementary Variants of Affine Configurational Theorems[1]

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Summary. We present elementary versions of Pappus, Major Desargues and Minor Desargues Axioms (i.e. statements formulated entirely in the language of points and parallelity of segments). Evidently they are consequences of appropriate configurational axioms introduced in the article [2]. In particular it follows that there exists an affine plane satisfying all of them.

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The articles [4], [1], [3], and [2] provide the notation and terminology for this paper.

In this paper S_1 denotes an affine plane.

We now state several propositions:

(1) Suppose S_1 satisfies **PAP**. Let a_1, a_2, a_3, b_1, b_2, b_3 be elements of S_1. If $a_1,a_2 \parallel a_1,a_3$ and $b_1,b_2 \parallel b_1,b_3$ and $a_1,b_2 \parallel a_2,b_1$ and $a_2,b_3 \parallel a_3,b_2$, then $a_3,b_1 \parallel a_1,b_3$.

(2) Suppose S_1 satisfies **DES**. Let o, a, a', b, b', c, c' be elements of S_1. Suppose $o,a \nparallel o,b$ and $o,a \nparallel o,c$ and $o,a \parallel o,a'$ and $o,b \parallel o,b'$ and $o,c \parallel o,c'$ and $a,b \parallel a',b'$ and $a,c \parallel a',c'$. Then $b,c \parallel b',c'$.

(3) Suppose S_1 satisfies **des**. Let a, a', b, b', c, c' be elements of S_1. Suppose $a,a' \nparallel a,b$ and $a,a' \nparallel a,c$ and $a,a' \parallel b,b'$ and $a,a' \parallel c,c'$ and $a,b \parallel a',b'$ and $a,c \parallel a',c'$. Then $b,c \parallel b',c'$.

(5)[1] There exists S_1 such that

(i) for all elements o, a, a', b, b', c, c' of S_1 such that $o,a \nparallel o,b$ and $o,a \nparallel o,c$ and $o,a \parallel o,a'$ and $o,b \parallel o,b'$ and $o,c \parallel o,c'$ and $a,b \parallel a',b'$ and $a,c \parallel a',c'$ holds $b,c \parallel b',c'$,

(ii) for all elements a, a', b, b', c, c' of S_1 such that $a,a' \nparallel a,b$ and $a,a' \nparallel a,c$ and $a,a' \parallel b,b'$ and $a,a' \parallel c,c'$ and $a,b \parallel a',b'$ and $a,c \parallel a',c'$ holds $b,c \parallel b',c'$,

(iii) for all elements a_1, a_2, a_3, b_1, b_2, b_3 of S_1 such that $a_1,a_2 \parallel a_1,a_3$ and $b_1,b_2 \parallel b_1,b_3$ and $a_1,b_2 \parallel a_2,b_1$ and $a_2,b_3 \parallel a_3,b_2$ holds $a_3,b_1 \parallel a_1,b_3$, and

(iv) for all elements a, b, c, d of S_1 such that $a,b \nparallel a,c$ and $a,b \parallel c,d$ and $a,c \parallel b,d$ holds $a,d \nparallel b,c$.

(6) Let o, a be elements of S_1. Then there exists an element p of S_1 such that for all elements b, c of S_1 holds

[1] Supported by RPBP.III-24.C2.

[1] The proposition (4) has been removed.

$o,a \upuparrows o,p$ and there exists an element d of S_1 such that if $o,p \upuparrows o,b$, then $o,c \upuparrows o,d$ and $p,c \upuparrows b,d$.

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