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On the Isomorphism between Finite Chains

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The articles [12], [10], [15], [13], [1], [3], [17], [14], [16], [6], [7], [4], [5], [11], [9], [8], and [2] provide the notation and terminology for this paper.

A relational structure is said to be a chain if:

(Def. 1) It is a connected non empty poset or it is empty.

Let us mention that every relational structure which is empty is also reflexive, transitive, and antisymmetric.

Let us note that every chain is reflexive, transitive, and antisymmetric.

Let us note that there exists a chain which is non empty.

Let us mention that every non empty chain is connected.

Let L be a 1-sorted structure. We say that L is countable if and only if:

(Def. 2) The carrier of L is countable.

Let us mention that there exists a chain which is finite and non empty.

Let us note that there exists a chain which is countable.

Let A be a connected non empty relational structure. Observe that every non empty relational substructure of A which is full is also connected.

Let A be a finite relational structure. One can verify that every relational substructure of A is finite.

One can prove the following proposition

(1) For all natural numbers n, m such that $n \leq m$ holds $\langle n, \subseteq \rangle$ is a full relational substructure of $\langle m, \subseteq \rangle$.

Let L be a relational structure and let A, B be sets. We say that A, B form upper lower partition of L if and only if:

(Def. 3) $A \cup B =$ the carrier of L and for all elements a, b of L such that $a \in A$ and $b \in B$ holds $a < b$.

One can prove the following propositions:

(2) Let L be a relational structure and A, B be sets. If A, B form upper lower partition of L, then A misses B.

(3) Let L be an upper-bounded antisymmetric non empty relational structure. Then (the carrier of L) $\setminus \{\top_L\}$, $\{\top_L\}$ form upper lower partition of L.

(4) Let L_1, L_2 be relational structures and f be a map from L_1 into L_2. Suppose f is isomorphic. Then

(i) the carrier of $L_1 \neq \emptyset$ iff the carrier of $L_2 \neq \emptyset$,

(ii) the carrier of $L_2 \neq \emptyset$ or the carrier of $L_1 = \emptyset$, and

(iii) the carrier of $L_1 = \emptyset$ iff the carrier of $L_2 = \emptyset$.

(5) Let L_1, L_2 be antisymmetric relational structures and A_1, B_1 be subsets of L_1. Suppose A_1, B_1 form upper lower partition of L_1. Let A_2, B_2 be subsets of L_2. Suppose A_2, B_2 form upper lower partition of L_2. Let f be a map from $\mathrm{sub}(A_1)$ into $\mathrm{sub}(A_2)$. Suppose f is isomorphic. Let g be a map from $\mathrm{sub}(B_1)$ into $\mathrm{sub}(B_2)$. Suppose g is isomorphic. Then there exists a map h from L_1 into L_2 such that $h = f{+\cdot}g$ and h is isomorphic.

(6) Let A be a finite chain and n be a natural number. If $\overline{\overline{\text{the carrier of } A}} = n$, then A and $\langle n, \subseteq \rangle$ are isomorphic.

References

[1] Grzegorz Bancerek. Cardinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/card_1.html.

[2] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal1.html.

[3] Grzegorz Bancerek. Countable sets and Hessenberg's theorem. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/card_4.html.

[4] Grzegorz Bancerek. Bounds in posets and relational substructures. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/yellow_0.html.

[5] Grzegorz Bancerek. Directed sets, nets, ideals, filters, and maps. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/waybel_0.html.

[6] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[7] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[8] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/funct_4.html.

[9] Czesław Byliński. Galois connections. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/waybel_1.html.

[10] Agata Darmochwał. Finite sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finset_1.html.

[11] Adam Grabowski and Robert Milewski. Boolean posets, posets under inclusion and products of relational structures. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/yellow_1.html.

[12] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[13] Andrzej Trybulec. Many-sorted sets. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/pboole.html.

[14] Wojciech A. Trybulec. Partially ordered sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/orders_1.html.

[15] Wojciech A. Trybulec. Groups. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/group_1.html.

[16] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

[17] Edmund Woronowicz and Anna Zalewska. Properties of binary relations. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_2.html.

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