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The Class of Series-Parallel Graphs. Part II[1]

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Summary. In this paper we introduce two new operations on graphs: sum and union corresponding to parallel and series operation respectively. We determine *N*-free graph as the graph that does not embed Necklace 4. We define "fin_RelStr" as the set of all graphs with finite carriers. We also define the smallest class of graphs which contains the one-element graph and which is closed under parallel and series operations. The goal of the article is to prove the theorem that the class of finite series-parallel graphs is the class of finite *N*-free graphs. This paper formalizes the next part of [12].

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The articles [14], [13], [18], [7], [20], [8], [1], [2], [3], [15], [17], [4], [16], [19], [11], [5], [6], [9], and [10] provide the notation and terminology for this paper.

In this paper U is a universal class.

Next we state two propositions:

(1) For all sets X, Y such that $X \in U$ and $Y \in U$ and for every relation R between X and Y holds $R \in U$.

(2) The internal relation of Necklace 4 $= \{\langle 0, 1\rangle, \langle 1, 0\rangle, \langle 1, 2\rangle, \langle 2, 1\rangle, \langle 2, 3\rangle, \langle 3, 2\rangle\}$.

Let n be a natural number. Note that every element of $\mathbf{R}_n$ is finite.

One can prove the following proposition

(3) For every set x such that $x \in \mathbf{U}_0$ holds x is finite.

One can verify that every element of $\mathbf{U}_0$ is finite.

Let us mention that every number which is finite and ordinal is also natural.

Let G be a non empty relational structure. We say that G is N-free if and only if:

(Def. 1) G does not embed Necklace 4.

Let us mention that there exists a non empty relational structure which is strict, finite, and N-free.

Let R, S be relational structures. The functor UnionOf(R,S) yielding a strict relational structure is defined by the conditions (Def. 2).

(Def. 2)(i) The carrier of UnionOf$(R,S) =$ (the carrier of R) $\cup$ (the carrier of S), and

(ii) the internal relation of UnionOf$(R,S) =$ (the internal relation of R) $\cup$ (the internal relation of S).

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Let R, S be relational structures. The functor SumOf(R,S) yields a strict relational structure and is defined by the conditions (Def. 3).

(Def. 3)(i) The carrier of SumOf(R,S) = (the carrier of R) $\cup$ (the carrier of S), and

(ii) the internal relation of SumOf(R,S) = (the internal relation of R) $\cup$ (the internal relation of S) $\cup$ [: the carrier of R, the carrier of S :] $\cup$ [: the carrier of S, the carrier of R :].

The functor FinRelStr is defined by the condition (Def. 4).

(Def. 4) Let X be a set. Then $X \in$ FinRelStr if and only if there exists a strict relational structure R such that $X = R$ and the carrier of $R \in \mathbf{U}_0$.

One can check that FinRelStr is non empty.
The subset FinRelStrSp of FinRelStr is defined by the conditions (Def. 5).

(Def. 5)(i) For every strict relational structure R such that the carrier of R is non empty and trivial and the carrier of $R \in \mathbf{U}_0$ holds $R \in$ FinRelStrSp,

(ii) for all strict relational structures H_1, H_2 such that the carrier of H_1 misses the carrier of H_2 and $H_1 \in$ FinRelStrSp and $H_2 \in$ FinRelStrSp holds UnionOf$(H_1,H_2) \in$ FinRelStrSp and SumOf$(H_1,H_2) \in$ FinRelStrSp, and

(iii) for every subset M of FinRelStr such that for every strict relational structure R such that the carrier of R is non empty and trivial and the carrier of $R \in \mathbf{U}_0$ holds $R \in M$ and for all strict relational structures H_1, H_2 such that the carrier of H_1 misses the carrier of H_2 and $H_1 \in M$ and $H_2 \in M$ holds UnionOf$(H_1,H_2) \in M$ and SumOf$(H_1,H_2) \in M$ holds FinRelStrSp $\subseteq M$.

Let us note that FinRelStrSp is non empty.
The following four propositions are true:

(4) For every set X such that $X \in$ FinRelStrSp holds X is a finite strict non empty relational structure.

(5) For every relational structure R such that $R \in$ FinRelStrSp holds the carrier of $R \in \mathbf{U}_0$.

(6) Let X be a set. Suppose $X \in$ FinRelStrSp. Then

(i) X is a strict non empty trivial relational structure, or

(ii) there exist strict relational structures H_1, H_2 such that the carrier of H_1 misses the carrier of H_2 and $H_1 \in$ FinRelStrSp and $H_2 \in$ FinRelStrSp and $X =$ UnionOf(H_1,H_2) or $X =$ SumOf(H_1,H_2).

(7) For every strict non empty relational structure R such that $R \in$ FinRelStrSp holds R is N-free.

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