JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

Three-Argument Operations and Four-Argument Operations[1]

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Summary. The article contains the definition of three- and four- argument operations. The article introduces also a few operation related schemes: *FuncEx3D*, *TriOpEx*, *Lambda3D*, *TriOpLambda*, *FuncEx4D*, *QuaOpEx*, *Lambda4D*, *QuaOpLambda*.

MML Identifier: MULTOP_1.

WWW: http://mizar.org/JFM/Vol2/multop_1.html

The articles [3], [6], [1], [2], [5], and [4] provide the notation and terminology for this paper.

Let f be a function and let a, b, c be sets. The functor $f(a,b,c)$ yields a set and is defined as follows:

(Def. 1) $f(a,b,c) = f(\langle a,b,c\rangle)$.

For simplicity, we use the following convention: A, B, C, D, E are non empty sets, a is an element of A, b is an element of B, c is an element of C, d is an element of D, and X, Y, Z, S, x, y, z, s are sets.

Let us consider A, B, C, D, let f be a function from $[:A,B,C:]$ into D, and let us consider a, b, c. Then $f(a,b,c)$ is an element of D.

Next we state three propositions:

(2)[1] Let f_1, f_2 be functions from $[:X,Y,Z:]$ into D. Suppose that for all x, y, z such that $x \in X$ and $y \in Y$ and $z \in Z$ holds $f_1(\langle x,y,z\rangle) = f_2(\langle x,y,z\rangle)$. Then $f_1 = f_2$.

(3) For all functions f_1, f_2 from $[:A,B,C:]$ into D such that for all a, b, c holds $f_1(\langle a,b,c\rangle) = f_2(\langle a,b,c\rangle)$ holds $f_1 = f_2$.

(4) Let f_1, f_2 be functions from $[:A,B,C:]$ into D. Suppose that for every element a of A and for every element b of B and for every element c of C holds $f_1(a,b,c) = f_2(a,b,c)$. Then $f_1 = f_2$.

Let A be a set. A ternary operation on A is a function from $[:A,A,A:]$ into A.

In this article we present several logical schemes. The scheme *FuncEx3D* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ and a 4-ary predicate $\mathcal{P}$, and states that:

> There exists a function f from $[:\mathcal{A},\mathcal{B},\mathcal{C}:]$ into $\mathcal{D}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ holds $\mathcal{P}[x,y,z,f(\langle x,y,z\rangle)]$

provided the following requirement is met:

[1] Supported by RPBP.III-24.C6.

[1] The proposition (1) has been removed.

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- Let x be an element of $\mathcal{A}$, y be an element of $\mathcal{B}$, and z be an element of $\mathcal{C}$. Then there exists an element t of $\mathcal{D}$ such that $\mathcal{P}[x,y,z,t]$.

The scheme *TriOpEx* deals with a non empty set $\mathcal{A}$ and a 4-ary predicate $\mathcal{P}$, and states that:

There exists a ternary operation o on $\mathcal{A}$ such that for all elements a, b, c of $\mathcal{A}$ holds $\mathcal{P}[a,b,c,o(a, b, c)]$

provided the following condition is met:

- For all elements x, y, z of $\mathcal{A}$ there exists an element t of $\mathcal{A}$ such that $\mathcal{P}[x,y,z,t]$.

The scheme *Lambda3D* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ and a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$, and states that:

There exists a function f from $[:\mathcal{A}, \mathcal{B}, \mathcal{C}:]$ into $\mathcal{D}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ holds $f(\langle x, y, z\rangle) = \mathcal{F}(x,y,z)$

for all values of the parameters.

The scheme *TriOpLambda* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ and a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{D}$, and states that:

There exists a function o from $[:\mathcal{A}, \mathcal{B}, \mathcal{C}:]$ into $\mathcal{D}$ such that for every element a of $\mathcal{A}$ and for every element b of $\mathcal{B}$ and for every element c of $\mathcal{C}$ holds $o(a, b, c) = \mathcal{F}(a,b,c)$

for all values of the parameters.

Let f be a function and let a, b, c, d be sets. The functor $f(a, b, c, d)$ yields a set and is defined as follows:

(Def. 2) $f(a, b, c, d) = f(\langle a,b,c,d\rangle)$.

Let us consider A, B, C, D, E, let f be a function from $[:A, B, C, D:]$ into E, and let us consider a, b, c, d. Then $f(a, b, c, d)$ is an element of E.

We now state three propositions:

(6)[2] Let f_1, f_2 be functions from $[:X, Y, Z, S:]$ into D. Suppose that for all x, y, z, s such that $x \in X$ and $y \in Y$ and $z \in Z$ and $s \in S$ holds $f_1(\langle x,y,z,s\rangle) = f_2(\langle x,y,z,s\rangle)$. Then $f_1 = f_2$.

(7) For all functions f_1, f_2 from $[:A, B, C, D:]$ into E such that for all a, b, c, d holds $f_1(\langle a,b,c,d\rangle) = f_2(\langle a,b,c,d\rangle)$ holds $f_1 = f_2$.

(8) For all functions f_1, f_2 from $[:A, B, C, D:]$ into E such that for all a, b, c, d holds $f_1(a, b, c, d) = f_2(a, b, c, d)$ holds $f_1 = f_2$.

Let us consider A. A quadrary operation on A is a function from $[:A, A, A, A:]$ into A.

Now we present four schemes. The scheme *FuncEx4D* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ and a 5-ary predicate $\mathcal{P}$, and states that:

There exists a function f from $[:\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}:]$ into $\mathcal{E}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ and for every element s of $\mathcal{D}$ holds $\mathcal{P}[x,y,z,s, f(\langle x,y,z,s\rangle)]$

provided the parameters meet the following requirement:

- Let x be an element of $\mathcal{A}$, y be an element of $\mathcal{B}$, z be an element of $\mathcal{C}$, and s be an element of $\mathcal{D}$. Then there exists an element t of $\mathcal{E}$ such that $\mathcal{P}[x,y,z,s,t]$.

The scheme *QuaOpEx* deals with a non empty set $\mathcal{A}$ and a 5-ary predicate $\mathcal{P}$, and states that:

There exists a quadrary operation o on $\mathcal{A}$ such that for all elements a, b, c, d of $\mathcal{A}$ holds $\mathcal{P}[a,b,c,d,o(a, b, c, d)]$

provided the parameters meet the following condition:

- For all elements x, y, z, s of $\mathcal{A}$ there exists an element t of $\mathcal{A}$ such that $\mathcal{P}[x,y,z,s,t]$.

The scheme *Lambda4D* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ and a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{E}$, and states that:

There exists a function f from $[:\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}:]$ into $\mathcal{E}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ and for every element z of $\mathcal{C}$ and for every element s of $\mathcal{D}$ holds $f(\langle x,y,z,s\rangle) = \mathcal{F}(x,y,z,s)$

[2] The proposition (5) has been removed.

for all values of the parameters.

The scheme *QuaOpLambda* deals with a non empty set $\mathcal{A}$ and a 4-ary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

> There exists a quadrary operation o on $\mathcal{A}$ such that for all elements a, b, c, d of $\mathcal{A}$ holds $o(a, b, c, d) = \mathcal{F}(a,b,c,d)$

for all values of the parameters.

References

[1] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[2] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[3] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[4] Andrzej Trybulec. Domains and their Cartesian products. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/domain_1.html.

[5] Andrzej Trybulec. Tuples, projections and Cartesian products. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/mcart_1.html.

[6] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

Received October 2, 1990

Published January 2, 2004