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Examples of Category Structures

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Summary. This article contains definitions of two category structures: the category of many sorted signatures and the category of many sorted algebras. Some facts about these structures are proved.

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The articles [16], [8], [21], [20], [22], [5], [6], [23], [7], [9], [1], [4], [15], [2], [17], [13], [18], [12], [19], [14], [11], [3], and [10] provide the notation and terminology for this paper.

1. Category of Many Sorted Signatures

In this paper A denotes a non empty set, S denotes a non void non empty many sorted signature, and x denotes a set.

Let us consider A. The functor $\mathrm{MSSCat}(A)$ yielding a strict non empty category structure is defined by the conditions (Def. 1).

(Def. 1)(i) The carrier of $\mathrm{MSSCat}(A) = \mathrm{MSS\text{-}set}(A)$,

(ii) for all elements i, j of $\mathrm{MSS\text{-}set}(A)$ holds (the arrows of $\mathrm{MSSCat}(A))(i, j) = \mathrm{MSS\text{-}morph}(i, j)$, and

(iii) for all objects i, j, k of $\mathrm{MSSCat}(A)$ such that $i \in \mathrm{MSS\text{-}set}(A)$ and $j \in \mathrm{MSS\text{-}set}(A)$ and $k \in \mathrm{MSS\text{-}set}(A)$ and for all functions f_1, f_2, g_1, g_2 such that $\langle f_1, f_2\rangle \in$ (the arrows of $\mathrm{MSSCat}(A))(i, j)$ and $\langle g_1, g_2\rangle \in$ (the arrows of $\mathrm{MSSCat}(A))(j, k)$ holds (the composition of $\mathrm{MSSCat}(A))(i, j, k)(\langle g_1, g_2\rangle, \langle f_1, f_2\rangle) = \langle g_1 \cdot f_1, g_2 \cdot f_2\rangle$.

Let us consider A. One can verify that $\mathrm{MSSCat}(A)$ is transitive and associative and has units.

We now state the proposition

(1) For every category C such that $C = \mathrm{MSSCat}(A)$ holds every object of C is a non empty non void many sorted signature.

Let us consider S. Note that there exists an algebra over S which is strict and feasible.

Let us consider S, A. The functor $\mathrm{MSAlg\_set}(S,A)$ is defined by the condition (Def. 2).

(Def. 2) $x \in \mathrm{MSAlg\_set}(S,A)$ if and only if there exists a strict feasible algebra M over S such that $x = M$ and for every component C of the sorts of M holds $C \subseteq A$.

Let us consider S, A. Note that $\mathrm{MSAlg\_set}(S,A)$ is non empty.

2. Category of Many Sorted Algebras

In the sequel o is an operation symbol of S.

Next we state four propositions:

(2) Let x be an algebra over S. Suppose $x \in \mathrm{MSAlg\_set}(S,A)$. Then the sorts of $x \in (2^A)^{\text{the carrier of } S}$ and the characteristics of $x \in ((\mathbb{N} \dot{\to} A) \dot{\to} A)^{\text{the operation symbols of } S}$.

(3) Let U_1, U_2 be algebras over S. Suppose the sorts of U_1 are transformable to the sorts of U_2 and $\mathrm{Args}(o,U_1) \neq \emptyset$. Then $\mathrm{Args}(o,U_2) \neq \emptyset$.

(4) Let U_1, U_2, U_3 be feasible algebras over S, F be a many sorted function from U_1 into U_2, G be a many sorted function from U_2 into U_3, and x be an element of $\mathrm{Args}(o,U_1)$. Suppose that

(i) $\mathrm{Args}(o,U_1) \neq \emptyset$,

(ii) the sorts of U_1 are transformable to the sorts of U_2, and

(iii) the sorts of U_2 are transformable to the sorts of U_3.

Then there exists a many sorted function G_1 from U_1 into U_3 such that $G_1 = G \circ F$ and $G_1 \# x = G \# (F \# x)$.

(5) Let U_1, U_2, U_3 be feasible algebras over S, F be a many sorted function from U_1 into U_2, and G be a many sorted function from U_2 into U_3. Suppose that

(i) the sorts of U_1 are transformable to the sorts of U_2,

(ii) the sorts of U_2 are transformable to the sorts of U_3,

(iii) F is a homomorphism of U_1 into U_2, and

(iv) G is a homomorphism of U_2 into U_3.

Then there exists a many sorted function G_1 from U_1 into U_3 such that $G_1 = G \circ F$ and G_1 is a homomorphism of U_1 into U_3.

Let us consider S, A and let i, j be sets. Let us assume that $i \in \mathrm{MSAlg\_set}(S,A)$ and $j \in \mathrm{MSAlg\_set}(S,A)$. The functor $\mathrm{MSAlg\_morph}(S,A,i,j)$ is defined by the condition (Def. 3).

(Def. 3) $x \in \mathrm{MSAlg\_morph}(S,A,i,j)$ if and only if there exist strict feasible algebras M, N over S and there exists a many sorted function f from M into N such that $M = i$ and $N = j$ and $f = x$ and the sorts of M are transformable to the sorts of N and f is a homomorphism of M into N.

Let us consider S, A. The functor $\mathrm{MSAlgCat}(S,A)$ yielding a strict non empty category structure is defined by the conditions (Def. 4).

(Def. 4)(i) The carrier of $\mathrm{MSAlgCat}(S,A) = \mathrm{MSAlg\_set}(S,A)$,

(ii) for all elements i, j of $\mathrm{MSAlg\_set}(S,A)$ holds (the arrows of $\mathrm{MSAlgCat}(S,A))(i, j) = \mathrm{MSAlg\_morph}(S,A,i,j)$, and

(iii) for all objects i, j, k of $\mathrm{MSAlgCat}(S,A)$ and for all function yielding functions f, g such that $f \in$ (the arrows of $\mathrm{MSAlgCat}(S,A))(i, j)$ and $g \in$ (the arrows of $\mathrm{MSAlgCat}(S,A))(j, k)$ holds (the composition of $\mathrm{MSAlgCat}(S,A))(i, j, k)(g, f) = g \circ f$.

Let us consider S, A. One can verify that $\mathrm{MSAlgCat}(S,A)$ is transitive and associative and has units.

We now state the proposition

(6) For every category C such that $C = \mathrm{MSAlgCat}(S,A)$ holds every object of C is a strict feasible algebra over S.

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