JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

N-Tuples and Cartesian Products for $n = 9$[1]

Michał Muzalewski
Warsaw University
Białystok

Wojciech Skaba
Nicolaus Copernicus University
Toruń

Summary. This article defines ordered n-tuples, projections and Cartesian products for $n = 9$. We prove many theorems concerning the basic properties of the n-tuples and Cartesian products that may be utilized in several further, more challenging applications. A few of these theorems are a strightforward consequence of the regularity axiom. The article originated as an upgrade of the article [7].

MML Identifier: MCART_6.

WWW: http://mizar.org/JFM/Vol2/mcart_6.html

The articles [6], [1], [8], [7], [2], [3], [4], and [5] provide the notation and terminology for this paper.

For simplicity, we adopt the following rules: x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 denote sets, y, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9 denote sets, X, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9 denote sets, Y, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11}, Y_{12}, Y_{13}, Y_{14}, Y_{15} denote sets, Z denotes a set, x_{10} denotes an element of X_1, x_{11} denotes an element of X_2, x_{12} denotes an element of X_3, x_{13} denotes an element of X_4, x_{14} denotes an element of X_5, x_{15} denotes an element of X_6, x_{16} denotes an element of X_7, and x_{17} denotes an element of X_8.

Next we state two propositions:

(1) Suppose $X \neq \emptyset$. Then there exists Y such that

(i) $Y \in X$, and

(ii) for all Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11}, Y_{12}, Y_{13}, Y_{14} such that $Y_1 \in Y_2$ and $Y_2 \in Y_3$ and $Y_3 \in Y_4$ and $Y_4 \in Y_5$ and $Y_5 \in Y_6$ and $Y_6 \in Y_7$ and $Y_7 \in Y_8$ and $Y_8 \in Y_9$ and $Y_9 \in Y_{10}$ and $Y_{10} \in Y_{11}$ and $Y_{11} \in Y_{12}$ and $Y_{12} \in Y_{13}$ and $Y_{13} \in Y_{14}$ and $Y_{14} \in Y$ holds Y_1 misses X.

(2) Suppose $X \neq \emptyset$. Then there exists Y such that

(i) $Y \in X$, and

(ii) for all Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11}, Y_{12}, Y_{13}, Y_{14}, Y_{15} such that $Y_1 \in Y_2$ and $Y_2 \in Y_3$ and $Y_3 \in Y_4$ and $Y_4 \in Y_5$ and $Y_5 \in Y_6$ and $Y_6 \in Y_7$ and $Y_7 \in Y_8$ and $Y_8 \in Y_9$ and $Y_9 \in Y_{10}$ and $Y_{10} \in Y_{11}$ and $Y_{11} \in Y_{12}$ and $Y_{12} \in Y_{13}$ and $Y_{13} \in Y_{14}$ and $Y_{14} \in Y_{15}$ and $Y_{15} \in Y$ holds Y_1 misses X.

Let us consider x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9. The functor $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$ is defined by:

(Def. 1) $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle = \langle \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \rangle, x_9 \rangle$.

One can prove the following propositions:

[1]Supported by RPBP.III-24.C6.

(3) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle = \langle\langle\langle\langle\langle\langle\langle\langle x_1, x_2\rangle, x_3\rangle, x_4\rangle, x_5\rangle, x_6\rangle, x_7\rangle, x_8\rangle, x_9\rangle$.

(5)[1] $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle = \langle\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle, x_8, x_9\rangle$.

(6) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle = \langle\langle x_1,x_2,x_3,x_4,x_5,x_6\rangle,x_7,x_8,x_9\rangle$.

(7) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle = \langle\langle x_1,x_2,x_3,x_4,x_5\rangle,x_6,x_7,x_8,x_9\rangle$.

(8) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle = \langle\langle x_1,x_2,x_3,x_4\rangle,x_5,x_6,x_7,x_8,x_9\rangle$.

(9) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle = \langle\langle x_1, x_2, x_3\rangle,x_4,x_5,x_6,x_7,x_8,x_9\rangle$.

(10) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle = \langle\langle x_1, x_2\rangle,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle$.

(11) If $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle = \langle y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8,y_9\rangle$, then $x_1 = y_1$ and $x_2 = y_2$ and $x_3 = y_3$ and $x_4 = y_4$ and $x_5 = y_5$ and $x_6 = y_6$ and $x_7 = y_7$ and $x_8 = y_8$ and $x_9 = y_9$.

Let us consider X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9. The functor $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!]$ yielding a set is defined as follows:

(Def. 2) $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: [\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8 :\!], X_9 :\!]$.

The following propositions are true:

(12) $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: [\!: [\!: [\!: [\!: [\!: [\!: [\!: X_1, X_2 :\!], X_3 :\!], X_4 :\!], X_5 :\!], X_6 :\!], X_7 :\!], X_8 :\!], X_9 :\!]$.

(14)[2] $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: [\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :\!], X_8, X_9 :\!]$.

(15) $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: [\!: X_1, X_2, X_3, X_4, X_5\ X_6 :\!], X_7, X_8, X_9 :\!]$.

(16) $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: [\!: X_1, X_2, X_3, X_4, X_5 :\!], X_6, X_7, X_8, X_9 :\!]$.

(17) $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: [\!: X_1, X_2, X_3, X_4 :\!], X_5, X_6, X_7, X_8\ X_9 :\!]$.

(18) $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: [\!: X_1, X_2, X_3 :\!], X_4, X_5, X_6, X_7\ X_8\ X_9 :\!]$.

(19) $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: [\!: X_1, X_2 :\!], X_3, X_4, X_5, X_6\ X_7\ X_8\ X_9 :\!]$.

(20) $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ iff $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] \neq \emptyset$.

(21) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$. Suppose $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7\ Y_8\ Y_9 :\!]$. Then $X_1 = Y_1$ and $X_2 = Y_2$ and $X_3 = Y_3$ and $X_4 = Y_4$ and $X_5 = Y_5$ and $X_6 = Y_6$ and $X_7 = Y_7$ and $X_8 = Y_8$ and $X_9 = Y_9$.

(22) Suppose $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] \neq \emptyset$ and $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!] = [\!: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7\ Y_8\ Y_9 :\!]$. Then $X_1 = Y_1$ and $X_2 = Y_2$ and $X_3 = Y_3$ and $X_4 = Y_4$ and $X_5 = Y_5$ and $X_6 = Y_6$ and $X_7 = Y_7$ and $X_8 = Y_8$ and $X_9 = Y_9$.

(23) If $[\!: X, X, X, X, X\ X\ X\ X\ X :\!] = [\!: Y, Y, Y, Y, Y\ Y\ Y\ Y\ Y :\!]$, then $X = Y$.

In the sequel x_{18} denotes an element of X_9.

Next we state the proposition

(24) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$. Let x be an element of $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!]$. Then there exist x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$.

Let us consider X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9. Let us assume that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$. Let x be an element of $[\!: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :\!]$. The functor $x_{\mathbf{1}}$ yielding an element of X_1 is defined by:

[1] The proposition (4) has been removed.
[2] The proposition (13) has been removed.

(Def. 3) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{1}} = x_1$.

The functor $x_{\mathbf{2}}$ yields an element of X_2 and is defined as follows:

(Def. 4) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{2}} = x_2$.

The functor $x_{\mathbf{3}}$ yields an element of X_3 and is defined by:

(Def. 5) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{3}} = x_3$.

The functor $x_{\mathbf{4}}$ yielding an element of X_4 is defined as follows:

(Def. 6) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{4}} = x_4$.

The functor $x_{\mathbf{5}}$ yields an element of X_5 and is defined as follows:

(Def. 7) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{5}} = x_5$.

The functor $x_{\mathbf{6}}$ yielding an element of X_6 is defined by:

(Def. 8) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{6}} = x_6$.

The functor $x_{\mathbf{7}}$ yields an element of X_7 and is defined as follows:

(Def. 9) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{7}} = x_7$.

The functor $x_{\mathbf{8}}$ yielding an element of X_8 is defined as follows:

(Def. 10) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{8}} = x_8$.

The functor $x_{\mathbf{9}}$ yielding an element of X_9 is defined by:

(Def. 11) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$, then $x_{\mathbf{9}} = x_9$.

Next we state several propositions:

(25) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :]$ and given x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9. Suppose $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle$. Then $x_{\mathbf{1}} = x_1$ and $x_{\mathbf{2}} = x_2$ and $x_{\mathbf{3}} = x_3$ and $x_{\mathbf{4}} = x_4$ and $x_{\mathbf{5}} = x_5$ and $x_{\mathbf{6}} = x_6$ and $x_{\mathbf{7}} = x_7$ and $x_{\mathbf{8}} = x_8$ and $x_{\mathbf{9}} = x_9$.

(26) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :]$. Then $x = \langle x_{\mathbf{1}}, x_{\mathbf{2}}, x_{\mathbf{3}}, x_{\mathbf{4}}, x_{\mathbf{5}}, x_{\mathbf{6}}, x_{\mathbf{7}}, x_{\mathbf{8}}, x_{\mathbf{9}} \rangle$.

(27) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :]$. Then $x_{\mathbf{1}} = ((((((((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}}$ and $x_{\mathbf{2}} = ((((((((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{3}} = (((((((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{4}} = ((((((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{5}} = (((((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{6}} = ((((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{7}} = (((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{8}} = ((x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{9}} = (x\ \mathbf{qua}\ \mathrm{set})_{\mathbf{2}}$.

(28) Suppose $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :]$ meets $[: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7\ Y_8\ Y_9 :]$. Then X_1 meets Y_1 and X_2 meets Y_2 and X_3 meets Y_3 and X_4 meets Y_4 and X_5 meets Y_5 and X_6 meets Y_6 and X_7 meets Y_7 and X_8 meets Y_8 and X_9 meets Y_9.

(29) $[: \{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}\ \{x_6\}\ \{x_7\}\ \{x_8\}\ \{x_9\} :] = \{\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \rangle\}$.

For simplicity, we follow the rules: A_1 is a subset of X_1, A_2 is a subset of X_2, A_3 is a subset of X_3, A_4 is a subset of X_4, A_5 is a subset of X_5, A_6 is a subset of X_6, A_7 is a subset of X_7, A_8 is a subset of X_8, A_9 is a subset of X_9, and x is an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :]$.

We now state a number of propositions:

(30) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$. Let given x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9. Suppose $x = \langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle$. Then $x_{\mathbf{1}} = x_1$ and $x_{\mathbf{2}} = x_2$ and $x_{\mathbf{3}} = x_3$ and $x_{\mathbf{4}} = x_4$ and $x_{\mathbf{5}} = x_5$ and $x_{\mathbf{6}} = x_6$ and $x_{\mathbf{7}} = x_7$ and $x_{\mathbf{8}} = x_8$ and $x_{\mathbf{9}} = x_9$.

(31) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_1 = x_{10}$. Then $y_1 = x_{\mathbf{1}}$.

(32) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_2 = x_{11}$. Then $y_2 = x_{\mathbf{2}}$.

(33) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_3 = x_{12}$. Then $y_3 = x_{\mathbf{3}}$.

(34) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_4 = x_{13}$. Then $y_4 = x_{\mathbf{4}}$.

(35) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_5 = x_{14}$. Then $y_5 = x_{\mathbf{5}}$.

(36) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_6 = x_{15}$. Then $y_6 = x_{\mathbf{6}}$.

(37) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_7 = x_{16}$. Then $y_7 = x_{\mathbf{7}}$.

(38) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_8 = x_{17}$. Then $y_8 = x_{\mathbf{8}}$.

(39) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and for all x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18} such that $x = \langle x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17},x_{18}\rangle$ holds $y_9 = x_{18}$. Then $y_9 = x_{\mathbf{9}}$.

(40) Suppose $y \in [:X_1, X_2, X_3, X_4, X_5\, X_6\, X_7\, X_8\, X_9:]$. Then there exist x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 such that $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$ and $x_5 \in X_5$ and $x_6 \in X_6$ and $x_7 \in X_7$ and $x_8 \in X_8$ and $x_9 \in X_9$ and $y = \langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle$.

(41) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle \in [:X_1, X_2, X_3, X_4, X_5\, X_6\, X_7\, X_8\, X_9:]$ iff $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$ and $x_5 \in X_5$ and $x_6 \in X_6$ and $x_7 \in X_7$ and $x_8 \in X_8$ and $x_9 \in X_9$.

(42) Suppose that for every y holds $y \in Z$ iff there exist x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 such that $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$ and $x_5 \in X_5$ and $x_6 \in X_6$ and $x_7 \in X_7$ and $x_8 \in X_8$ and $x_9 \in X_9$ and $y = \langle x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\rangle$. Then $Z = [:X_1, X_2, X_3, X_4, X_5\, X_6\, X_7\, X_8\, X_9:]$.

(43) Suppose that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $X_8 \neq \emptyset$ and $X_9 \neq \emptyset$ and $Y_1 \neq \emptyset$ and $Y_2 \neq \emptyset$ and $Y_3 \neq \emptyset$ and $Y_4 \neq \emptyset$ and $Y_5 \neq \emptyset$ and $Y_6 \neq \emptyset$ and $Y_7 \neq \emptyset$ and $Y_8 \neq \emptyset$ and $Y_9 \neq \emptyset$. Let x be an element of $[:X_1, X_2, X_3, X_4, X_5\, X_6\, X_7\, X_8\, X_9:]$ and y be an element of $[:Y_1, Y_2, Y_3, Y_4, Y_5\, Y_6\, Y_7\, Y_8\, Y_9:]$. Suppose $x = y$. Then $x_{\mathbf{1}} = y_{\mathbf{1}}$ and $x_{\mathbf{2}} = y_{\mathbf{2}}$ and $x_{\mathbf{3}} = y_{\mathbf{3}}$ and $x_{\mathbf{4}} = y_{\mathbf{4}}$ and $x_{\mathbf{5}} = y_{\mathbf{5}}$ and $x_{\mathbf{6}} = y_{\mathbf{6}}$ and $x_{\mathbf{7}} = y_{\mathbf{7}}$ and $x_{\mathbf{8}} = y_{\mathbf{8}}$ and $x_{\mathbf{9}} = y_{\mathbf{9}}$.

(44) Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :]$. Suppose $x \in [: A_1, A_2, A_3, A_4, A_5\ A_6\ A_7\ A_8\ A_9 :]$. Then $x_{\mathbf{1}} \in A_1$ and $x_{\mathbf{2}} \in A_2$ and $x_{\mathbf{3}} \in A_3$ and $x_{\mathbf{4}} \in A_4$ and $x_{\mathbf{5}} \in A_5$ and $x_{\mathbf{6}} \in A_6$ and $x_{\mathbf{7}} \in A_7$ and $x_{\mathbf{8}} \in A_8$ and $x_{\mathbf{9}} \in A_9$.

(45) If $X_1 \subseteq Y_1$ and $X_2 \subseteq Y_2$ and $X_3 \subseteq Y_3$ and $X_4 \subseteq Y_4$ and $X_5 \subseteq Y_5$ and $X_6 \subseteq Y_6$ and $X_7 \subseteq Y_7$ and $X_8 \subseteq Y_8$ and $X_9 \subseteq Y_9$, then $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :] \subseteq [: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7\ Y_8\ Y_9 :]$.

(46) $[: A_1, A_2, A_3, A_4, A_5\ A_6\ A_7\ A_8\ A_9 :]$ is a subset of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7\ X_8\ X_9 :]$.

REFERENCES

[1] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/zfmisc_1.html`.

[2] Michał Muzalewski and Wojciech Skaba. n-tuples and Cartesian products for $n = 5$. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/mcart_2.html`.

[3] Michał Muzalewski and Wojciech Skaba. n-tuples and Cartesian products for $n = 6$. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/mcart_3.html`.

[4] Michał Muzalewski and Wojciech Skaba. n-tuples and Cartesian products for $n = 7$. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/mcart_4.html`.

[5] Michał Muzalewski and Wojciech Skaba. n-tuples and Cartesian products for $n = 8$. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/mcart_5.html`.

[6] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[7] Andrzej Trybulec. Tuples, projections and Cartesian products. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/mcart_1.html`.

[8] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

Received October 15, 1990

Published January 2, 2004