Calculation of Matrices of Field Elements. Part I

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Summary.   This article gives property of calculation of matrices.

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The articles [8], [5], [10], [11], [2], [1], [3], [4], [13], [6], [7], [12], and [9] provide the notation and terminology for this paper.

In this paper i denotes a natural number.

Let K be a field and let M_1, M_2 be matrices over K. The functor $M_1 - M_2$ yielding a matrix over K is defined as follows:

(Def. 1) $M_1 - M_2 = M_1 + -M_2$.

Next we state a number of propositions:

(1)   For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ holds $--M = M$.

(2)   For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ holds $M + -M = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}^{(\operatorname{len} M) \times (\operatorname{width} M)}_{K}$.

(3)   For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ holds $M - M = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}^{(\operatorname{len} M) \times (\operatorname{width} M)}_{K}$.

(4)   Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_1 + M_3 = M_2 + M_3$. Then $M_1 = M_2$.

(5)   For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_2 > 0$ holds $M_1 - -M_2 = M_1 + M_2$.

(6)   For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_1 = M_1 + M_2$ holds $M_2 = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}_{K}$.

(7) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and

$\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_2 = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}_K$ holds

$M_1 = M_2$.

(8) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and

$\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_1 + M_2 = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}_K$ holds

$M_2 = -M_1$.

(9) For all natural numbers n, m and for every field K such that $n > 0$ holds

$$-\begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}^{n \times m}_K = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}^{n \times m}_K.$$

(10) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_2 - M_1 = M_2$ holds $M_1 = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}_K$.

(11) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = M_1 - (M_2 - M_2)$.

(12) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $-(M_1 + M_2) = -M_1 + -M_2$.

(13) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 - (M_1 - M_2) = M_2$.

(14) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_3 = M_2 - M_3$. Then $M_1 = M_2$.

(15) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_3 - M_1 = M_3 - M_2$. Then $M_1 = M_2$.

(16) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - M_2 - M_3 = M_1 - M_3 - M_2$.

(17) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - M_3 = M_1 - M_2 - (M_3 - M_2)$.

(18) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_3 - M_1 - (M_3 - M_2) = M_2 - M_1$.

(19) Let K be a field and M_1, M_2, M_3, M_4 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{len} M_3 = \operatorname{len} M_4$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{width} M_3 = \operatorname{width} M_4$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_2 = M_3 - M_4$. Then $M_1 - M_3 = M_2 - M_4$.

(20) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = M_1 + (M_2 - M_2)$.

(21) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = (M_1 + M_2) - M_2$.

(22) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = (M_1 - M_2) + M_2$.

(23) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 + M_3 = M_1 + M_2 + (M_3 - M_2)$.

(24) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $(M_1 + M_2) - M_3 = (M_1 - M_3) + M_2$.

(25) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $(M_1 - M_2) + M_3 = (M_3 - M_2) + M_1$.

(26) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 + M_3 = (M_1 + M_2) - (M_2 - M_3)$.

(27) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - M_3 = (M_1 + M_2) - (M_3 + M_2)$.

(28) Let K be a field and M_1, M_2, M_3, M_4 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{len} M_3 = \operatorname{len} M_4$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{width} M_3 = \operatorname{width} M_4$ and $\operatorname{len} M_1 > 0$ and $M_1 + M_2 = M_3 + M_4$. Then $M_1 - M_3 = M_4 - M_2$.

(29) Let K be a field and M_1, M_2, M_3, M_4 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{len} M_3 = \operatorname{len} M_4$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{width} M_3 = \operatorname{width} M_4$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_3 = M_4 - M_2$. Then $M_1 + M_2 = M_3 + M_4$.

(30) Let K be a field and M_1, M_2, M_3, M_4 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{len} M_3 = \operatorname{len} M_4$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{width} M_3 = \operatorname{width} M_4$ and $\operatorname{len} M_1 > 0$ and $M_1 + M_2 = M_3 - M_4$. Then $M_1 + M_4 = M_3 - M_2$.

(31) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - (M_2 + M_3) = M_1 - M_2 - M_3$.

(32) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - (M_2 - M_3) = (M_1 - M_2) + M_3$.

(33) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - (M_2 - M_3) = M_1 + (M_3 - M_2)$.

(34) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_1 - M_3 = (M_1 - M_2) + (M_2 - M_3)$.

(35) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $-M_1 = -M_2$, then $M_1 = M_2$.

(36) For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ and $-M = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}_K^{(\operatorname{len} M) \times (\operatorname{width} M)}$ holds $M = \begin{pmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{pmatrix}_K^{(\operatorname{len} M) \times (\operatorname{width} M)}$.

(37) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ and $M_1 + -M_2 = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}_K^{(\operatorname{len} M_1) \times (\operatorname{width} M_1)}$ holds $M_1 = M_2$.

(38) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = M_1 + M_2 + -M_2$.

(39) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = M_1 + (M_2 + -M_2)$.

(40) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = -M_2 + M_1 + M_2$.

(41) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $-(-M_1 + M_2) = M_1 + -M_2$.

(42) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 + M_2 = -(-M_1 + -M_2)$.

(43) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $-(M_1 - M_2) = M_2 - M_1$.

(44) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $-M_1 - M_2 = -M_2 - M_1$.

(45) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = -M_2 - (-M_1 - M_2)$.

(46) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $-M_1 - M_2 - M_3 = -M_1 - M_3 - M_2$.

(47) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $-M_1 - M_2 - M_3 = -M_2 - M_3 - M_1$.

(48) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $-M_1 - M_2 - M_3 = -M_3 - M_2 - M_1$.

(49) Let K be a field and M_1, M_2, M_3 be matrices over K. If $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$, then $M_3 - M_1 - (M_3 - M_2) = -(M_1 - M_2)$.

(50) For every field K and for every matrix M over K such that $\operatorname{len} M > 0$ holds $\begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}_K^{(\operatorname{len} M) \times (\operatorname{width} M)} - M = -M$.

(51) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 + M_2 = M_1 - -M_2$.

(52) For every field K and for all matrices M_1, M_2 over K such that $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{len} M_1 > 0$ holds $M_1 = M_1 - (M_2 + -M_2)$.

(53) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_1 - M_3 = M_2 + -M_3$. Then $M_1 = M_2$.

(54) Let K be a field and M_1, M_2, M_3 be matrices over K. Suppose $\operatorname{len} M_1 = \operatorname{len} M_2$ and $\operatorname{len} M_2 = \operatorname{len} M_3$ and $\operatorname{width} M_1 = \operatorname{width} M_2$ and $\operatorname{width} M_2 = \operatorname{width} M_3$ and $\operatorname{len} M_1 > 0$ and $M_3 - M_1 = M_3 + -M_2$. Then $M_1 = M_2$.

(55) Let K be a field and A, B be matrices over K. If $\operatorname{len} A = \operatorname{len} B$ and $\operatorname{width} A = \operatorname{width} B$, then the indices of A = the indices of B.

(56) Let K be a field and x, y, z be finite sequences of elements of the carrier of K. If $\operatorname{len} x = \operatorname{len} y$ and $\operatorname{len} y = \operatorname{len} z$, then $(x + y) \bullet z = x \bullet z + y \bullet z$.

(57) Let K be a field and x, y, z be finite sequences of elements of the carrier of K. If $\operatorname{len} x = \operatorname{len} y$ and $\operatorname{len} y = \operatorname{len} z$, then $z \bullet (x + y) = z \bullet x + z \bullet y$.

(58) Let D be a non empty set and M be a matrix over D. Suppose $\operatorname{len} M > 0$. Let n be a natural number. Then M is a matrix over D of dimension $n \times \operatorname{width} M$ if and only if $n = \operatorname{len} M$.

(59) Let K be a field, j be a natural number, and A, B be matrices over K. Suppose $\operatorname{len} A = \operatorname{len} B$ and $\operatorname{width} A = \operatorname{width} B$ and there exists a natural number j such that $\langle i, j \rangle \in$ the indices of A. Then $\operatorname{Line}(A + B, i) = \operatorname{Line}(A, i) + \operatorname{Line}(B, i)$.

(60) Let K be a field, j be a natural number, and A, B be matrices over K. Suppose $\operatorname{len} A = \operatorname{len} B$ and $\operatorname{width} A = \operatorname{width} B$ and there exists a natural number i such that $\langle i, j \rangle \in$ the indices of A. Then $(A + B)_{\Box, j} = A_{\Box, j} + B_{\Box, j}$.

(61) Let V_1 be a field and P_1, P_2 be finite sequences of elements of the carrier of V_1. If $\operatorname{len} P_1 = \operatorname{len} P_2$, then $\sum (P_1 + P_2) = \sum P_1 + \sum P_2$.

(62) Let K be a field and A, B, C be matrices over K. If $\operatorname{len} B = \operatorname{len} C$ and $\operatorname{width} B = \operatorname{width} C$ and $\operatorname{width} A = \operatorname{len} B$ and $\operatorname{len} A > 0$ and $\operatorname{len} B > 0$, then $A \cdot (B + C) = A \cdot B + A \cdot C$.

(63) Let K be a field and A, B, C be matrices over K. If $\operatorname{len} B = \operatorname{len} C$ and $\operatorname{width} B = \operatorname{width} C$ and $\operatorname{len} A = \operatorname{width} B$ and $\operatorname{len} B > 0$ and $\operatorname{len} A > 0$, then $(B + C) \cdot A = B \cdot A + C \cdot A$.

(64) Let K be a field, n, m, k be natural numbers, M_1 be a matrix over K of dimension $n \times m$, and M_2 be a matrix over K of dimension $m \times k$. Suppose $\operatorname{width} M_1 = \operatorname{len} M_2$ and $0 < \operatorname{len} M_1$ and $0 < \operatorname{len} M_2$. Then $M_1 \cdot M_2$ is a matrix over K of dimension $n \times k$.

REFERENCES

[1] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finseq_1.html.

[2] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[3] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[4] Czesław Byliński. Binary operations applied to finite sequences. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/finseqop.html.

[5] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/finseq_2.html.

[6] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/matrix_1.html.

[7] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/vectsp_1.html.

[8] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[9] Wojciech A. Trybulec. Vectors in real linear space. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/rlvect_1.html.

[10] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[11] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

[12] Katarzyna Zawadzka. Sum and product of finite sequences of elements of a field. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/fvsum_1.html`.

[13] Katarzyna Zawadzka. The product and the determinant of matrices with entries in a field. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/matrix_3.html`.